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Geometría Analítica II

Grupo: 4080

Trabajo 21

Se tiene la cuádrica escrita

$$(x+y-z)^2+(x-2y)^2+(z+4y)^2=2013$$

1) Escribirla en forma matricial

$\to x^2+2xy-2xz+y^2-2yz+z^2+x^2-4xy+4y^2+z^2+8yz+16y^2-2013=0$

$$2x^2+21y^2+2z^2-2xy-2xz+6yz-2013=0.$$

$$\to A=\begin{pmatrix}2&-1&-1\\-1&21&3\\-1&3&2\end{pmatrix}\qquad \gamma=-2013$$

$$\to \boxed{Q(p)=(x\ y\ z)\begin{pmatrix}2&-1&-1\\-1&21&3\\-1&3&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}-2013}$$

2) Aplicar el teorema espectral.

Para el teorema espectral necesitamos encontrar una matriz Q y D de tal manera que:

$AQ=QD$ con D diagonal y Q formada por columnas con los vectores propios y D con los valores propios

$$\to \begin{pmatrix}2&-1&-1\\-1&21&3\\-1&3&2\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}-\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=0$$

$$\begin{pmatrix}2-\lambda&-1&-1\\-1&21-\lambda&3\\-1&3&2-\lambda\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=0$$

Hallando las soluciones de "λ", resolviendo el determinante.

$$(2-\lambda)(21-\lambda)(2-\lambda)+(-1)(-1)(3)+(-1)(3)(-1)-(-1)(21-\lambda)(-1)-(3)(3)(2-\lambda)$$

$$=(42-23\lambda+\lambda^2)(2-\lambda)+3+3-21+\lambda-18+9\lambda-2+\lambda$$

$$=84-42\lambda-46\lambda+23\lambda^2+2\lambda^2-\lambda^3+6-21+\lambda-18+9\lambda-2+\lambda$$

$$=-\lambda^3+25\lambda^2-77\lambda+49=0$$

→ Buscando una raíz por Método de Newton

$$f(\lambda)=-\lambda^3+25\lambda^2-77\lambda+49 \qquad \rightsquigarrow\ x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$f'(\lambda)=-3\lambda^2+50\lambda-77$$

$x_0=1$

$$x_1=1-\frac{-4}{-30}=0.8666\ \to\ x_1=0.8666$$

$$x_2=0.8666-\left(\frac{0.4194}{-35.949}\right)=0.8776\ \to\ x_2=0.8776$$

$$x_3=0.8776-\left(\frac{0.00343}{-35.430}\right)=0.8776\ \to\ \boxed{\lambda_1=0.8776}$$

→ Hallando "λ_2" y "λ_3" con división sintética

$$\begin{array}{r|l} & -\lambda^2 + 24.122\lambda - 55.831 \\ \lambda - 0.8776 & -\lambda^3 + 25\lambda^2 - 77\lambda + 49 \\ & \underline{\lambda^3 - 0.8776\lambda^2} \\ & 0 \quad 24.122\lambda^2 - 77\lambda \\ & \underline{-24.122\lambda^2 + 21.169\lambda} \\ & 0 \quad -55.831\lambda + 49 \\ & \underline{55.831\lambda - 49} \\ & 0 \end{array}$$

Resolviendo el cociente

$-\lambda^2 + 24.122\lambda - 55.831$; por fórmula general

$$\lambda = \frac{-24.122 \pm \sqrt{581.87 - 4(55.831)}}{-2}$$

$$\lambda_2 = \frac{-24.122 + 18.935}{-2} \qquad \lambda_3 = \frac{-24.122 - 18.935}{-2}$$

$$\boxed{\lambda_2 = 2.593} \qquad \boxed{\lambda_3 = 21.528}$$

$$\begin{cases} \lambda_1 = 0.8776 \\ \lambda_2 = -2.593 \\ \lambda_3 = 21.528 \end{cases} \rightarrow \text{Vectores propios}$$

VECTORES PROPIOS

- Para $\lambda_1 = 0.8776$

$$\begin{pmatrix} 2-0.8776 & -1 & -1 \\ -1 & 21-0.8776 & 3 \\ -1 & 3 & 2-0.8776 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1.1224 & -1 & -1 \\ -1 & 20.1224 & 3 \\ -1 & 3 & 1.1224 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0. \rightarrow \begin{cases} 1.1224u_1 - u_2 - u_3 = 0 & (1) \\ -u_1 + 20.1224u_2 + 3u_3 = 0 & (2) \\ -u_1 + 3u_2 + 1.1224u_3 = 0 & (3) \end{cases}$$

- Sea $u_2 = 1$ y sumando (1) y (3)

$$\begin{array}{l} 1.1224u_1 - u_3 = 1 \\ \underline{-u_1 + 1.1224u_3 = -3} \\ 0.1224u_1 + 0.1224u_3 = -2. \quad (4) \end{array}$$

- Sumando (1) y (2)

$$\begin{array}{l} 1.1224u_1 - u_3 = 1 \\ \underline{-u_1 + 3u_3 = -20.1224} \\ 0.1224u_1 + 2u_3 = -19.1224 \end{array}$$

$$\rightarrow \begin{cases} \cancel{0.1224u_1} + 0.1224u_3 = -2 \quad (-1) \\ \underline{\cancel{0.1224u_1} + 2u_3 = -19.1224} \end{cases}$$

$$1.8776u_3 = -17.1224$$

$$u_3 = -9.119$$

Buscando "u_1" en (1)

$$1.1224u_1 - 1 + 9.119$$

$$1.1224u_1 = -8.119$$

$$u_1 = -7.233$$

↪ Luego obtengamos $\|u\|$ para normalizar los vectores que son los que interesan

$$\|u\| = \sqrt{(-7.233)^2 + 1 + (-9.119)^2}$$

$$\|u\| = \sqrt{136.47245}$$

$$\|u\| = 11.682$$

$$\rightarrow \frac{1}{11.682} \cdot \begin{pmatrix} -7.233 \\ 1 \\ -9.119 \end{pmatrix} = u \rightarrow u = \begin{pmatrix} -0.619 \\ 0.085 \\ -0.780 \end{pmatrix}$$

• Para $\lambda_2 = 2.593$

$$\begin{pmatrix} -0.593 & -1 & -1 \\ -1 & 18.407 & 3 \\ -1 & 3 & -0.593 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \rightarrow \begin{cases} -0.593 V_1 - V_2 - V_3 = 0 & (1) \\ -1 V_1 + 18.407 V_2 + 3 V_3 = 0 & (2) \\ -V_1 + 3 V_2 - 0.593 V_3 = 0 & (3) \end{cases}$$

• Sea $V_3 = 1$, sumando (1) y (2)

$$-0.593 V_1 - V_2 = 1$$
$$-V_1 + 18.407 V_2 = -3$$
$$-1.593 V_1 + 17.407 V_2 = -2 \quad (4)$$

• Sumando (1) y (3)

$$-0.593 V_1 - V_2 = 1$$
$$-V_1 + 3 V_2 = 0.593$$
$$-1.593 V_1 + 2 V_2 = 1.593 \quad (5)$$

De (4) y (5)

$$\rightarrow \begin{cases} -1.593 V_1 + 17.407 V_2 = -2 & (-1) \\ -1.593 V_1 + 2 V_2 = 1.593 \\ -15.407 V_2 = 3.593 \end{cases} \rightarrow V_2 = -0.233$$

Obteniendo V_1 en (1)

$$-0.593 V_1 + 0.233 - 1 = 0$$
$$V_1 = \frac{1 - 0.233}{-0.593} \rightarrow V_1 = -1.293$$

→ Obteniendo $\|V\|$ y después normalizando cada entrada

$$\|V\| = \sqrt{(-1.293)^2 + (-0.233)^2 + 1}$$
$$= \sqrt{2.7261}$$
$$= 1.651$$

$$\rightarrow \frac{1}{1.651} \begin{pmatrix} -1.293 \\ -0.233 \\ 1 \end{pmatrix} = V \rightarrow V = \begin{pmatrix} -0.783 \\ -0.141 \\ 0.605 \end{pmatrix}$$

• Para $\lambda_3 = 21.528$

$$\begin{pmatrix} -19.528 & -1 & -1 \\ -1 & -0.528 & 3 \\ -1 & 3 & -19.528 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \rightarrow \begin{cases} -19.528 \omega_1 - \omega_2 - \omega_3 = 0 & (1) \\ -\omega_1 - 0.528 \omega_2 + 3\omega_3 = 0 & (2) \\ -\omega_1 + 3\omega_2 - 19.528 \omega_3 = 0 & (3) \end{cases}$$

• Sea $\omega_3 = 1$, sumando (1) y (2)

$$-19.528 \omega_1 - \omega_2 = 1$$
$$-\omega_1 - 0.528 \omega_2 = -3$$
$$-20.528 \omega_1 - 1.528 \omega_2 = -2. \quad (4)$$

• Sumando (1) y (3)

$$-19.528 \omega_1 - \omega_2 = 1$$
$$-\omega_1 + 3\omega_2 = 19.528$$
$$-20.528 \omega_1 + 2\omega_2 = 20.528 \quad (5)$$

De (4) y (5)

$$\begin{cases} -20.528\,\omega_1 - 1.528\,\omega_2 = -2 & (-1) \\ -20.528\,\omega_1 + 2\omega_2 = 20.528 \\ \hline 3.528\,\omega_2 = 22.528 \end{cases} \rightarrow \underline{\omega_2 = 6.385}$$

→ Obteniendo ω_1 en (1)

$$-19.528\,\omega_1 + 6.385 - 1 = 0$$

$$\omega_1 = \frac{1+6.385}{-19.528} \rightarrow \underline{\omega_1 = -0.378}$$

Luego obtengamos $\|\omega\|$ para normalizar...

$$\|\omega\| = \sqrt{(-0.378)^2 + (6.385)^2 + 1}$$

$$= \sqrt{41.911}$$

$$\underline{\|\omega\| = 6.473}$$

$$\rightarrow \frac{1}{6.473}\begin{pmatrix} -0.378 \\ 6.385 \\ 1 \end{pmatrix} = \omega$$

$$\rightarrow \omega = \begin{pmatrix} -0.058 \\ 0.986 \\ 0.154 \end{pmatrix}$$

Entonces:

$$Q = \begin{pmatrix} u \,|\, v \,|\, \omega \end{pmatrix} = \begin{pmatrix} -0.619 & -0.783 & -0.058 \\ 0.085 & -0.141 & 0.986 \\ -0.780 & 0.605 & 0.154 \end{pmatrix}$$

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 0.8776 & 0 & 0 \\ 0 & 2.593 & 0 \\ 0 & 0 & 21.528 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix}\begin{pmatrix} -0.619 & -0.783 & -0.058 \\ 0.085 & -0.141 & 0.986 \\ -0.780 & 0.605 & 0.154 \end{pmatrix} = \begin{pmatrix} -0.619 & -0.783 & -0.058 \\ 0.085 & -0.141 & 0.986 \\ -0.780 & 0.605 & 0.154 \end{pmatrix}\begin{pmatrix} 0.8776 & 0 & 0 \\ 0 & 2.593 & 0 \\ 0 & 0 & 21.528 \end{pmatrix}$$

o bien

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} -0.619 & -0.783 & -0.058 \\ 0.085 & -0.141 & 0.986 \\ -0.780 & 0.605 & 0.154 \end{pmatrix}\begin{pmatrix} -0.619 & 0.085 & -0.780 \\ -0.783 & -0.141 & 0.605 \\ -0.058 & 0.986 & 0.154 \end{pmatrix}\begin{pmatrix} 0.8776 & 0 & 0 \\ 0 & 2.593 & 0 \\ 0 & 0 & 21.528 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1.998 & -0.99 & -0.99 \\ -0.99 & 20.98 & 2.98 \\ -0.99 & 2.98 & 1.99 \end{pmatrix}$$

→ Caja donde se encuentra la cuádrica

$$2x^2 + 21y^2 + 2z^2 - 2xy - 2xz + 6yz = 2013$$

→ $2x^2 = 2013$

$x^2 = \pm\sqrt{\frac{2013}{2}}$ → $\boxed{x = \pm\ 31.725}$

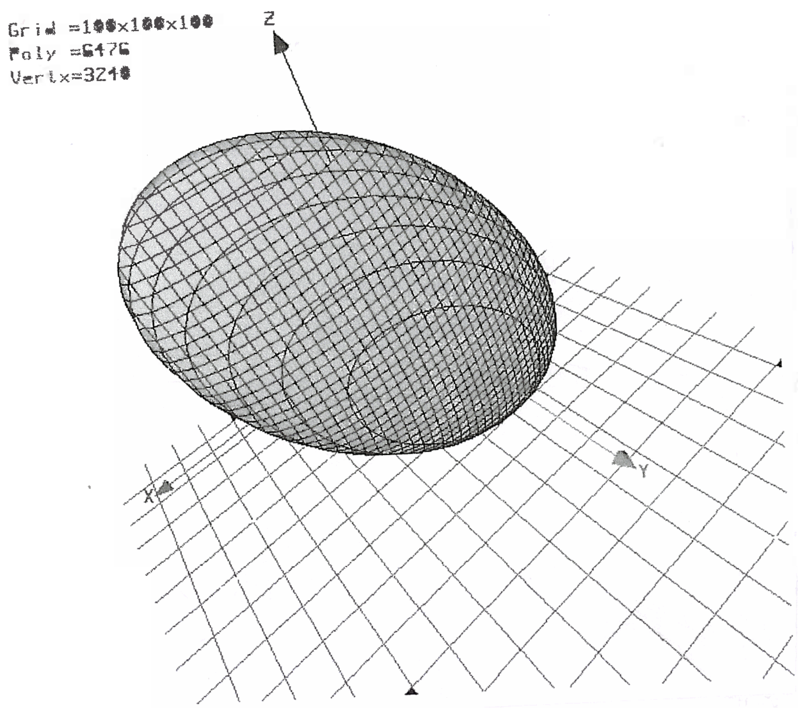
→ $21y^2 = 2013$

$y^2 = \pm\sqrt{\frac{2013}{21}}$ → $\boxed{y = \pm\ 9.79}$

→ $2z^2 = 2013$

$z^2 = \pm\sqrt{\frac{2013}{2}}$ → $\boxed{z = \pm\ 31.725}$

→ Gráfica de la cuádrica



La gráfica resultante es un elipsoide

Problema 2 Dada $A = \begin{pmatrix} -3 & 3 & 4 \\ 3 & 3 & 2 \\ 4 & 2 & -4 \end{pmatrix}$ y $\lambda_1 = -2$, $\lambda_2 = 5.557$, $\lambda_3 = -7.557$

$$\rightarrow A - \lambda u = \begin{pmatrix} -3-\lambda & 3 & 4 \\ 3 & 3-\lambda & 2 \\ 4 & 2 & -4-\lambda \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

• Para $\lambda_1 = -2$

$$\begin{pmatrix} -1 & 3 & 4 \\ 3 & 5 & 2 \\ 4 & 2 & -2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0 \rightarrow \begin{cases} -u_1 + 3u_2 + 4u_3 = 0 & (1) \\ 3u_1 + 5u_2 + 2u_3 = 0 & (2) \\ 4u_1 + 2u_2 - 2u_3 = 0 & (3) \end{cases}$$

• Tomemos $u_2 = 1$ y sumemos (1) y (2)

$$-u_1 + 4u_3 = -3$$
$$3u_1 + 2u_3 = -5$$
$$2u_1 + 6u_3 = -8 \quad (4)$$

• Sumando (1) y (3)

$$-u_1 + 4u_3 = -3$$
$$4u_1 - 2u_3 = -2$$
$$3u_1 + 2u_3 = -5 \quad (5)$$

$$\rightarrow \begin{cases} 2u_1 + 6u_3 = -8 \\ 3u_1 + 2u_3 = -5 \quad \cdot(-3) \end{cases}$$
$$-7u_1 = 7$$
$$u_1 = -1$$

• Hallando "u_3"

$$-3 + 5 + 2u_3 = 0$$
$$2u_3 = -2$$
$$u_3 = -1$$

$$\rightarrow u = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

• Para $\lambda_2 = 5.557$

$$\begin{pmatrix} -8.557 & 3 & 4 \\ 3 & -2.557 & 2 \\ 4 & 2 & -9.557 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = 0 \rightarrow \begin{cases} -8.557w_1 + 3w_2 + 4w_3 = 0 \\ 3w_1 - 2.557w_2 + 2w_3 = 0 \\ 4w_1 + 2w_2 - 1.557w_3 = 0 \end{cases}$$

• Tomemos $w_3 = 1$ y sumemos (1) y (2)

$$-8.557w_1 + 3w_2 = -4$$
$$3w_1 - 2.557w_2 = -2$$
$$-5.557w_1 + 0.443w_2 = -6$$

• Sumando (1) y (3)

$$-8.557w_1 + 3w_2 = -4$$
$$4w_1 + 2w_2 = 9.557$$
$$-4.557w_1 + 5w_2 = 5.557$$

$$\rightarrow \begin{cases} -5.557w_1 + 0.443w_2 = -6 & \cdot -5 \\ -4.557w_1 + 5w_2 = 5.557 & \cdot (0.443) \end{cases}$$
$$27.785w_1 - 2.215 = 30$$
$$-2.018w_1 + 2.215 = 2.461$$
$$25.767w_1 = 32.461$$
$$w_1 = 1.259$$

→ Hallando $\underline{W_2}$

$$4(1{,}259) + 2W_2 - 9{,}557 = 0$$
$$2W_2 = 9.557 - 5{,}036$$
$$w_2 = \frac{4{,}521}{2} \rightarrow \underline{W_2 = 2{,}260}$$

$$\rightarrow W = \frac{1}{2{,}773}\begin{pmatrix} 1.259 \\ 2.260 \\ 1 \end{pmatrix}$$

• Para $\lambda_3 = -7{,}557$

$$\begin{pmatrix} 4.557 & 3 & 4 \\ 3 & 10.557 & 2 \\ 4 & 2 & 3.557 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \rightarrow \begin{cases} 4.557V_1 + 3V_2 + 4V_3 = 0 \\ 3V_1 + 10.557V_2 + 2V_3 = 0 \\ 4V_1 + 2V_2 + 3.557V_3 = 0 \end{cases}$$

Tomemos $V_3 = 1$ y sumemos ① y ②

$$4.557V_1 + 3V_2 = -4$$
$$3V_1 + 10.557V_2 = -2$$
$$7.557V_1 + 13.557V_2 = -6$$

• Sumando ① y ③

$$4.557V_1 + 3V_2 = -4$$
$$4V_1 + 2V_2 = -3.557$$
$$8.557V_1 + 5V_2 = -7.557$$

$$\rightarrow \begin{cases} 7.557V_1 + 13.557V_2 = -6 & \times -5 \\ 8.557 + 5V_2 = -7.557 & (13.557) \\ -37.785V_1 - 67.785V_2 = 30 \\ 116.007V_1 + 67.785V_2 = -102.450 \end{cases}$$

$$78.222V_1 = -72.450$$
$$\underline{V_1 = -0.926}$$

• Hallando V_2

$$4.557(-0.926) + 3V_2 = -4$$
$$3V_2 = -4 + 4.219$$
$$\underline{V_2 = 0.073}$$

$$\rightarrow V = \frac{1}{1.364}\begin{pmatrix} -0.926 \\ 0.073 \\ 1 \end{pmatrix}$$