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Materia: Geometría Analítica II.

17 de Abril de 2013.

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Trabajo 21.

* **Problema:** Se tiene la cuádrica escrita en la forma: $Q(x,y,z) = (x+y-z)^2 + (x-2y)^2 + (z+4y)^2 = 2013.$

$\boxed{1}$. Escribir la cuádrica en forma matricial: $Q(P) = p^t A p + \gamma = 0.$

* Desarrollando $Q(x,y,z)$ tenemos:

$$x^2 + y^2 + z^2 + 2xy - 2xz - 2yz + x^2 - 4xy + 4y^2 + z^2 + 8yz + 16y^2 = 2013.$$

$$Q(x,y,z): 2x^2 + 21y^2 + 2z^2 - 2xy - 2xz + 6yz - 2013 = 0.$$

* Entonces: $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\gamma = -2013.$

$$\longrightarrow \boxed{Q(P) = p^t \begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix} p - 2013.}$$

$\boxed{2}$. Aplicar el teorema espectral.

* Como A es simétrica, entonces existen matrices Q ortogonal y D diagonal tales que

$$\boxed{AQ = DQ}$$

* Para encontrar D obtenemos los valores propios: $Au = \lambda u.$

$$(A - \lambda I)u = 0.$$

$$\left[\begin{pmatrix} 2 & -1 & -1 \\ -1 & 21 & 3 \\ -1 & 3 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right] u = \begin{pmatrix} 2-\lambda & -1 & -1 \\ -1 & 21-\lambda & 3 \\ -1 & 3 & 2-\lambda \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0.$$

* Resolvemos $\det(A - \lambda I) = 0$:

$$\begin{vmatrix} 2-\lambda & -1 & -1 \\ -1 & 21-\lambda & 3 \\ -1 & 3 & 2-\lambda \end{vmatrix} = (2-\lambda)(21-\lambda)(2-\lambda) + 3 + 3 - (21-\lambda) - 9(2-\lambda) - (2-\lambda)$$

$$= (42 - 23\lambda + \lambda^2)(2-\lambda) + 6 - 21 + \lambda - 18 + 9\lambda - 2 + \lambda$$

$$= 84 - 42\lambda - 46\lambda + 23\lambda^2 + 2\lambda^2 - \lambda^3 - 35 + 11\lambda$$

$$= -\lambda^3 + 25\lambda^2 - 77\lambda + 49 = 0.$$

* Resolvemos λ con <u>Método de Newton:</u> $-\lambda^3+25\lambda^2-77\lambda+49=0$

$\rightarrow f(x)=-x^3+25x^2-77x+49=0$

$\quad \hookrightarrow f'(x)=-3x^2+50x-77$

$$\boxed{x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}}$$

$\boxed{x_0=0} \rightarrow f(x_0)=49,\ f'(x_0)=-77 \rightarrow x_1=0-\left(\frac{49}{-77}\right) \rightarrow \boxed{x_1=0.6363}$

$f(x_1)=9.8692,\ f'(x_1)=-46.3996 \rightarrow x_2=0.6363-\left(\frac{9.8692}{-46.3996}\right) \rightarrow \boxed{x_2=0.8490}$

$f(x_2)=1.0350,\ f'(x_2)=-36.7124 \rightarrow x_3=0.849-\left(\frac{1.0350}{-36.7124}\right) \rightarrow \boxed{x_3=0.8771}$

$f(x_3)=0.0211,\ f'(x_3)=-35.4529 \rightarrow x_4=0.8771-\left(\frac{0.0211}{-35.4529}\right) \rightarrow \boxed{x_4=0.8776}$ <u>RAÍZ</u>

$$
\begin{array}{r|l}
 & -x^2+24.123x-55.84 \\
\hline
x-0.877 & -x^3+25x^2-77x+49 \\
 & x^3-0.877x^2 \\
\hline
 & 24.123x^2-77x \\
 & -24.123x^2+21.15x \\
\hline
 & -55.84x+49 \\
 & 55.84x-49 \\
\hline
 & 0
\end{array}
$$

$\rightarrow$ $\underline{-x^2+24.123x-55.84}$

$\hookrightarrow x=-24.123\pm\sqrt{581.91-4(55.84)}$

$x=\dfrac{-24.123\pm18.93}{-2} \rightarrow \boxed{\begin{array}{l}x_1=2.593\\ x_2=21.529\end{array}}$

$\rightarrow$ Por lo tanto tenemos 3 valores para λ:

$$\boxed{\begin{array}{l}\lambda_1=0.877\\ \lambda_2=2.593\\ \lambda_3=21.529\end{array}}$$ <u>VALORES PROPIOS.</u>

* Sustituyendo en $(A-\lambda I)u=0$:

$\boxed{\lambda_1}$: $\begin{pmatrix}2-0.877 & -1 & -1\\ -1 & 21-0.877 & 3\\ -1 & 3 & 2-0.877\end{pmatrix}=\begin{pmatrix}1.123 & -1 & -1\\ -1 & 20.123 & 3\\ -1 & 3 & 1.123\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=0$

$\hookrightarrow$
$1.123u_1-u_2-u_3=0$ (1)
$-u_1+20.123u_2+3u_3=0$ (2)
$-u_1+3u_2+1.123u_3=0$ (3)

* Sea $u_3=1$, y resolviendo el sistema:

$$\begin{array}{rl} 1.123u_1 - u_2 = 1 & (1) \\ -u_1 + 20.123u_2 = -3 & (2) \\ \hline 0.123u_1 + 19.123u_2 = -2 & (4) \end{array} \qquad \begin{array}{rl} 1.123u_1 - u_2 = 1 & (1) \\ -u_1 + 3u_2 = -1.123 & (3) \\ \hline 0.123u_1 + 2u_2 = -0.123 & (5) \end{array}$$

$$\hookrightarrow \begin{array}{rl} 0.123u_1 + 19.123u_2 = -2 & (4) \\ -0.123u_1 - 2u_2 = 0.123 & (5)(-1) \\ \hline 17.123u_2 = -1.877 & \end{array}$$

$$\boxed{\begin{array}{l} u_2 = -0.1096 \\ u_1 = 0.7795 \\ u_3 = 1 \end{array}} \longrightarrow \boxed{u = \begin{pmatrix} 0.7795 \\ -0.1096 \\ 1 \end{pmatrix}}$$

$\boxed{\lambda_2}$:

$$\begin{pmatrix} 2-2.593 & -1 & -1 \\ -1 & 21-2.593 & 3 \\ -1 & 3 & 2-2.593 \end{pmatrix} = \begin{pmatrix} -0.593 & -1 & -1 \\ -1 & 18.407 & 3 \\ -1 & 3 & -0.593 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0.$$

$$\hookrightarrow \begin{array}{rl} -0.593v_1 - v_2 - v_3 = 0 & (1) \\ -v_1 + 18.407v_2 + 3v_3 = 0 & (2) \\ -v_1 + 3v_2 - 0.593v_3 = 0 & (3) \end{array} \longrightarrow \text{Sea } v_3=1: \begin{array}{rl} -0.593v_1 - v_2 = 1 & (1) \\ -v_1 + 18.407v_2 = -3 & (2) \\ -v_1 + 3v_2 = 0.593 & (3) \end{array}$$

$$\hookrightarrow \begin{array}{rl} -\cancel{1.593v_1} + 17.407v_2 = -2 & (4) \longrightarrow (1)+(2) \\ \cancel{1.593v_1} - 2v_2 = -1.593 & (5) \longrightarrow [(1)+(3)](-1). \\ \hline 15.407v_2 = -3.593 & \end{array}$$

$$\boxed{\begin{array}{l} v_2 = -0.233 \\ v_1 = -1.293 \\ v_3 = 1 \end{array}} \longrightarrow \boxed{v = \begin{pmatrix} -1.293 \\ -0.233 \\ 1 \end{pmatrix}}$$

$\boxed{\lambda_3}$:

$$\begin{pmatrix} 2-21.529 & -1 & -1 \\ -1 & 21-21.529 & 3 \\ -1 & 3 & 2-21.529 \end{pmatrix} = \begin{pmatrix} -19.529 & -1 & -1 \\ -1 & -0.529 & 3 \\ -1 & 3 & -19.529 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = 0.$$

$$\hookrightarrow \begin{array}{rl} -19.529w_1 - w_2 - w_3 = 0 & (1) \\ -w_1 - 0.529w_2 + 3w_3 = 0 & (2) \\ -w_1 + 3w_2 - 19.529w_3 = 0 & (3) \end{array} \longrightarrow \text{Sea } w_3=1: \begin{array}{rl} -19.529w_1 - w_2 = 1 & (1) \\ -w_1 - 0.529w_2 = -3 & (2) \\ -w_1 + 3w_2 = 19.529 & (3) \end{array}$$

$$\hookrightarrow \begin{array}{rl} -\cancel{20.529w_1} - 1.529w_2 = -2 & (4) \longrightarrow (1)+(2) \\ \cancel{20.529w_1} - 2w_2 = -20.529. & (5) \longrightarrow [(1)+(3)](-1). \\ \hline -3.529w_2 = -22.529 & \end{array}$$

$$\boxed{w_2 = 6.383, \quad w_1 = -0.377, \quad w_3 = 1} \longrightarrow \boxed{w = \begin{pmatrix} -0.377 \\ 6.383 \\ 1 \end{pmatrix}}$$

* Por lo tanto:

$$\boxed{u=\begin{pmatrix}0.7795\\-0.1096\\1\end{pmatrix},\quad v=\begin{pmatrix}-1.293\\-0.233\\1\end{pmatrix},\quad w=\begin{pmatrix}-0.377\\6.383\\1\end{pmatrix}}$$

VECTORES PROPIOS.

* Normalizando cada vector:

$$u\cdot\frac{1}{\|u\|}=\frac{1}{1.2726}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\boxed{\begin{pmatrix}0.612\\-0.0861\\0.7857\end{pmatrix}}$$

$$v\cdot\frac{1}{\|v\|}=\frac{1}{1.6511}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\boxed{\begin{pmatrix}-0.7831\\-0.141\\0.605\end{pmatrix}}$$

$$w\cdot\frac{1}{\|w\|}=\frac{1}{6.4718}\begin{pmatrix}w_1\\w_2\\w_3\end{pmatrix}=\boxed{\begin{pmatrix}-0.058\\0.986\\0.154\end{pmatrix}}$$

* Ahora, con los vectores normalizados formamos la matriz ortogonal $Q=(\vec{u}\,|\,\vec{v}\,|\,\vec{w})$.

$$Q=\begin{pmatrix}0.612 & -0.7831 & -0.058\\-0.0861 & -0.141 & 0.986\\0.7857 & 0.605 & 0.154\end{pmatrix}$$

* Con los valores propios formamos la matriz diagonal D:

$$D=\begin{pmatrix}0.877 & 0 & 0\\0 & 2.593 & 0\\0 & 0 & 21.529\end{pmatrix}$$

* Finalmente, aplicando el teorema espectral:

$$\boxed{A=Q^tDQ}$$

$$\boxed{\begin{pmatrix}2 & -1 & -1\\-1 & 21 & 3\\-1 & 3 & 2\end{pmatrix}=\begin{pmatrix}0.612 & -0.0861 & 0.7857\\-0.7831 & -0.141 & 0.605\\-0.058 & 0.986 & 0.154\end{pmatrix}\begin{pmatrix}0.877 & 0 & 0\\0 & 2.593 & 0\\0 & 0 & 21.529\end{pmatrix}\begin{pmatrix}0.612 & -0.7831 & -0.058\\-0.0861 & -0.141 & 0.986\\0.7857 & 0.605 & 0.154\end{pmatrix}}$$

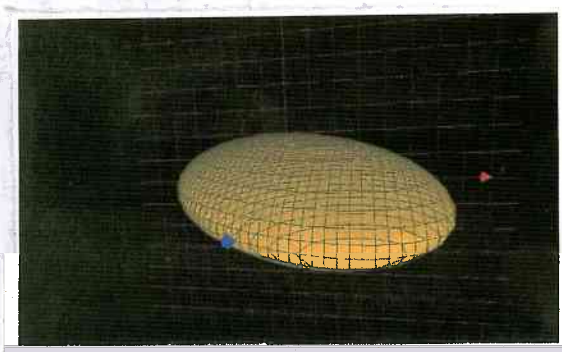
$\boxed{3}$. Graficar la cuádrica.

* $Q(x,y,z)$: $2x^2+21y^2+2z^2-2xy-2xz+6yz-2013=0$

$\longrightarrow 2x^2=2013 \longrightarrow x=\pm 31.72$, si $y,z=0$.

$\longrightarrow 21y^2=2013 \longrightarrow y=\pm 9.79$, si $x,z=0$.

$\longrightarrow 2z^2=2013 \longrightarrow z=\pm 31.72$, si $x,y=0$.



* **PROBLEMA 2:** Encontrar vectores propios de $B=\begin{pmatrix}-3 & 3 & 4\\ 3 & 3 & 2\\ 4 & 2 & -4\end{pmatrix}$, cuyo polinomio característico es $-\lambda^3-4\lambda^2+38\lambda+84$ y cuyos valores propios son: $\lambda_1=5.557$, $\lambda_2=-2$, $\lambda_3=-7.557$.

$\rightarrow$ $(B-\lambda I)u=0.$

$\boxed{\lambda_1}$

$$\begin{pmatrix}-3-5.557 & 3 & 4\\ 3 & 3-5.557 & 2\\ 4 & 2 & -4-5.557\end{pmatrix}=\begin{pmatrix}-8.557 & 3 & 4\\ 3 & -2.557 & 2\\ 4 & 2 & -9.557\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=0.$$

$$\begin{aligned}&① \quad -8.557u_1+3u_2+4u_3=0\\ &② \quad 3u_1-2.557u_2+2u_3=0\\ &③ \quad 4u_1+2u_2-9.557u_3=0.\end{aligned}$$

$\rightarrow$ Sea $u_1=1$:

$$\begin{aligned}&3u_2+4u_3=8.557 \quad ①\\ &-2.557u_2+2u_3=-3 \quad ②\\ &2u_2-9.557u_3=-4. \quad ③\end{aligned}$$

$$\begin{aligned}&5u_2-5.557u_3=4.557 \quad ④\\ &-0.557u_2-7.557u_3=-7. \quad ⑤\end{aligned}$$

$$\begin{aligned}&\cancel{u_2}-1.1114u_3=0.9114\\ &-\cancel{u_2}-13.5673u_3=-12.5673\end{aligned}$$

$$-14.6787u_3=-11.6559$$

$$\boxed{\begin{aligned}u_3&=0.7940\\ u_2&=1.793\\ u_1&=1\end{aligned}} \longrightarrow \boxed{u=\begin{pmatrix}1\\1.793\\0.794\end{pmatrix}}$$

$\boxed{\lambda_2}$

$$\begin{pmatrix}-3+2 & 3 & 4\\ 3 & 3+2 & 2\\ 4 & 2 & -4+2\end{pmatrix}=\begin{pmatrix}-1 & 3 & 4\\ 3 & 5 & 2\\ 4 & 2 & -2\end{pmatrix}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=0$$

$$\begin{aligned}&-v_1+3v_2+4v_3=0 \quad ①\\ &3v_1+5v_2+2v_3=0 \quad ②\\ &4v_1+2v_2-2v_3=0. \quad ③\end{aligned}$$

$\rightarrow$ Sea $v_1=1$:

$$\begin{aligned}&3v_2+4v_3=1 \quad ①\\ &5v_2+2v_3=-3 \quad ②\\ &2v_2-2v_3=-4. \quad ③\end{aligned}$$

$$\begin{aligned}&5v_2+2v_3=-3 \quad ④\\ &8v_2+6v_3=-2 \quad ⑤\end{aligned}$$

$$\begin{aligned}&-15v_2-\cancel{6v_3}=9\\ &8v_2+\cancel{6v_3}=-2.\end{aligned}$$

$$-7v_2=7$$

$$\boxed{\begin{aligned}v_2&=-1\\ v_3&=1\\ v_1&=1\end{aligned}} \longrightarrow \boxed{v=\begin{pmatrix}1\\-1\\1\end{pmatrix}}$$

$$\boxed{\lambda_3}: \begin{pmatrix} -3+7.557 & 3 & 4 \\ 3 & 3+7.557 & 2 \\ 4 & 2 & -4+7.557 \end{pmatrix} = \begin{pmatrix} 4.557 & 3 & 4 \\ 3 & 10.557 & 2 \\ 4 & 2 & 3.557 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = 0$$

$$\begin{aligned} 4.557 w_1 + 3 w_2 + 4 w_3 &= 0 \quad (1) \\ 3 w_1 + 10.557 w_2 + 2 w_3 &= 0 \quad (2) \\ 4 w_1 + 2 w_2 + 3.557 w_3 &= 0 \quad (3) \end{aligned}$$

$\rightarrow$ Sea $w_1 = 1$:

$$\begin{aligned} 3 w_2 + 4 w_3 &= -4.557 \quad (1) \\ 10.557 w_2 + 2 w_3 &= -3 \quad (2) \\ 2 w_2 + 3.557 w_3 &= -4 \quad (3) \end{aligned}$$

$$\begin{aligned} 13.557 w_2 + 6 w_3 &= -7.557 \quad (4) \\ 5 w_2 + 7.557 w_3 &= -8.557 \quad (5) \end{aligned}$$

$$\begin{aligned} \cancel{w_2} + 0.4425 w_3 &= -0.5574 \\ -\cancel{w_2} - 1.5114 w_3 &= 1.7114 \\ \hline -1.0689 w_3 &= 1.154 \end{aligned}$$

$$\boxed{\begin{aligned} w_3 &= -1.0796 \\ w_2 &= -0.0796 \\ w_1 &= 1 \end{aligned}} \longrightarrow$$

* Por lo tanto:

$$\boxed{u = \begin{pmatrix} 1 \\ 1.793 \\ 0.794 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} 1 \\ -0.0796 \\ -1.0796 \end{pmatrix}}$$

VECTORES PROPIOS.