Geometría Analítica I

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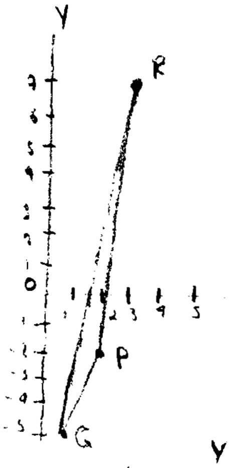
~ Trabajo 26 ~

Problema 1



ΔPQR formado por $P(2,-2)$, $Q(1,-5)$ y $R(3,7)$

$$\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix} = \begin{pmatrix}9 & 0\\ 0 & 2/5\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\hat{P} = \begin{pmatrix}9 & 0\\ 0 & 2/5\end{pmatrix}\begin{pmatrix}2\\ -2\end{pmatrix} = \begin{pmatrix}9(2)+0(-2)\\ 0(2)+\frac{2}{5}(-2)\end{pmatrix} = \begin{pmatrix}18\\ -\frac{4}{5}\end{pmatrix}$$

$$\tilde{Q} = \begin{pmatrix}9 & 0\\ 0 & 2/5\end{pmatrix}\begin{pmatrix}1\\ -5\end{pmatrix} = \begin{pmatrix}9(1)+0(-5)\\ 0(1)+\frac{2}{5}(-5)\end{pmatrix} = \begin{pmatrix}9\\ -2\end{pmatrix}$$

$$\tilde{R} = \begin{pmatrix}9 & 0\\ 0 & 2/5\end{pmatrix}\begin{pmatrix}3\\ 7\end{pmatrix} = \begin{pmatrix}9(3)+0(7)\\ 0(3)+\frac{2}{5}(7)\end{pmatrix} = \begin{pmatrix}27\\ 14/5\end{pmatrix}$$

POSICIÓN ~

Triángulo $\Delta\tilde{P}\tilde{Q}\tilde{R}$
$\hat{P}\,(18, -4/5)$
$\hat{Q}\,(9, -2)$
$\tilde{R}\,(27, 14/5)$

* De acuerdo a los puntos obtenidos, se puede concluir que los vectores $\overrightarrow{OP}$, $\overrightarrow{OQ}$ y $\overrightarrow{OR}$ (vectores del origen a los respectivos puntos), en sus componentes 'x' sufrieron una ampliación (9:1) y las componentes "y" una reducción (1: 2/5). Es decir, bajo el mismo sistema coordenado con origen O, los puntos $\vec{P}$, $\vec{Q}$ y $\vec{R}$ fueron trasladados a nuevos puntos, bajo la matriz de transformación dada, a los puntos $\hat{P}$, $\hat{Q}$ y $\tilde{R}$.

Problema 2

Triángulo $\Delta P_0P_1P_2$ con $P_0(-3,4)$, $P_1(5,3)$ y $P_2(-1,1)$

$$\tilde{P} = AP \Rightarrow \begin{pmatrix}\tilde{x}\\ \tilde{y}\end{pmatrix} = \begin{pmatrix}-4 & 8\\ 2 & -3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

$A \begin{pmatrix}-4 & 8\\ 2 & -3\end{pmatrix}$

$\Delta\tilde{P}_0\tilde{P}_1\tilde{P}_2$ POSICIÓN ~ $\tilde{P}_0(44,-18)$, $\tilde{P}_1(12,-3)$, $\tilde{P}_2(12,-5)$

$$\hat{P}_0 = \begin{pmatrix}-4 & 8\\ 2 & -3\end{pmatrix}\begin{pmatrix}-3\\ 4\end{pmatrix} = \begin{pmatrix}(-4)(-3)+8(4)\\ 2(-3)+\ldots(4)\end{pmatrix} = \begin{pmatrix}44\\ -18\end{pmatrix}$$

$$\tilde{P}_1 = \begin{pmatrix}-4 & 8\\ 2 & -3\end{pmatrix}\begin{pmatrix}3\\ 3\end{pmatrix} = \begin{pmatrix}-4(3)+8(3)\\ 2(3)+(-3)(3)\end{pmatrix} = \begin{pmatrix}12\\ -3\end{pmatrix}$$

$$\tilde{P}_2 = \begin{pmatrix}-4 & 8\\ 2 & -3\end{pmatrix}\begin{pmatrix}-1\\ 1\end{pmatrix} = \begin{pmatrix}-4(-1)+8(1)\\ 2(-1)+(-3)(1)\end{pmatrix} = \begin{pmatrix}12\\ -5\end{pmatrix}$$

$$\det A = \begin{vmatrix}-4 & 8\\ 2 & -3\end{vmatrix} = 12-16 = \boxed{-4} \qquad |\det A| = 4$$

(1) $\Delta P_0P_1P_2$ ÁREA

$$\text{Proy}_{\overrightarrow{P_1P_0}}\overrightarrow{P_1P_2} = \frac{\overrightarrow{P_1P_2}\cdot\overrightarrow{P_1P_0}}{\|\overrightarrow{P_1P_0}\|^2} = \frac{(-4,-2)\cdot(-6,1)}{36+1} = \frac{22}{37}(-6,1) = \left(-\frac{132}{37}, \frac{22}{37}\right) \Rightarrow \overrightarrow{QP_2} = (-4,-2)-\left(-\frac{132}{37}, \frac{22}{37}\right)$$

$$\overrightarrow{QP_2} = \left(-\frac{16}{37}, -\frac{96}{37}\right) \quad \|\overrightarrow{QP_2}\| = \frac{16\sqrt{37}}{37} \quad \|\overrightarrow{P_1P_0}\| = \sqrt{37} \quad A = \frac{\frac{16\sqrt{37}}{37}\cdot\sqrt{37}}{2} = \boxed{8\,u^2}$$

(2) $\Delta\hat{P}_0\hat{P}_1\hat{P}_2$

$$\text{Proy}_{\overrightarrow{\hat{P}_1\hat{P}_0}}\overrightarrow{\hat{P}_1\hat{P}_2} = \frac{\hat{P}_1\hat{P}_2\cdot\hat{P}_1\hat{P}_0}{\|\overrightarrow{\hat{P}_1\hat{P}_0}\|} = \frac{(0,-2)\cdot(32,-15)}{1024+225} = \frac{30}{1249}(32,-15) = \left(\frac{960}{1249}, -\frac{450}{1249}\right) \quad A = \frac{\frac{\sqrt{4096}}{\sqrt{1249}}\cdot\sqrt{1249}}{2}$$

$$\overrightarrow{\tilde{Q}\tilde{P}_2} = (0,-2)-\left(\frac{960}{1249}, -\frac{450}{1249}\right) = \left(-\frac{960}{1249}, -\frac{2048}{1249}\right) \quad \|\hat{Q}\hat{P}_2\| = \sqrt{\frac{4096}{1249}} \quad \|\overrightarrow{\hat{P}_1\hat{P}_0}\| = \sqrt{1249} \quad \boxed{32\,u^2}$$

∴ Se puede asociar que el área del triángulo nuevo es igual al área del triángulo original por el $|\det A| \Rightarrow Ar\,\Delta\hat{P}_0\hat{P}_1\hat{P}_2 = |\det A|\,(Ar\,\Delta P_0P_1P_2)$

Problema 3

Calcular la matriz inversa.

1) $A = \begin{pmatrix}4 & 3\\ -2 & 1\end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A}A_{ct}$

A_{ct} = Matriz de cofactores transpuesta

$\det A$ = Determinante de A

$A = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix} \Rightarrow A_c = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} \Rightarrow A_c^t = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$ $\det A = \begin{vmatrix} 4 & 3 \\ -2 & 1 \end{vmatrix} = 4-(-6) = 10$

$A^{-1} = \frac{1}{10} \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1/10 & -3/10 \\ 1/5 & 2/5 \end{pmatrix}$ $\boxed{A^{-1} = \begin{pmatrix} 1/10 & -3/10 \\ 1/5 & 2/5 \end{pmatrix}}$ $\boxed{A \cdot A^{-1} = I_A}$

2)

$B = \begin{pmatrix} 3 & 4 \\ 4 & -2 \end{pmatrix} \Rightarrow B_c = \begin{pmatrix} -2 & -4 \\ -4 & 3 \end{pmatrix} \Rightarrow B_c^t = \begin{pmatrix} -2 & -4 \\ -4 & 3 \end{pmatrix}$ $\det B = \begin{vmatrix} 3 & 4 \\ 4 & -2 \end{vmatrix} = -6-16 = -22$

$A^{-1} = -\frac{1}{22} \begin{pmatrix} -2 & -4 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 1/11 & 2/11 \\ 2/11 & -3/22 \end{pmatrix}$ $\boxed{B^{-1} = \begin{pmatrix} 1/11 & 2/11 \\ 2/11 & -3/22 \end{pmatrix}}$ $\boxed{B \cdot B^{-1} = I_B}$