THE MOUNTAIN CLIMBING THEOREM AND INVERSE LIMITS WITH SET VALUED FUNCTIONS

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Abstract. We will explore in detail several different ways to use the Mountain Climbing Theorem with inverse limits with set valued functions when those functions send points in the interval $[0,1]$ to closed subsets of $[0,1]$. In particular we will discuss how to show that certain topology spaces cannot be obtained as such an inverse limit, we will discuss the dynamics of the shift map when there is only one bonding function, and we will discuss applications to reversible properties of inverse limits, that is properties that if true for an inverse limit with a single set valued bonding function f are also true for the inverse limit with f^{-1}.

1. Exercises

E1 Let $G = \{(x,x) \in I^2 \mid x \in [0,1]\} \cup \{(x,0) \in I^2 \mid x \in [0,1]\} \cup \{(1,y) \in I^2 \mid y \in [0,1]\}$. Show that $\bigstar_{i=1}^{n} G$ is homeomorphic to K_{n+2}, the complete graph with $n+2$ vertices.

E2 A compact subset M of the Hilbert cube, I^∞, has the crossover property if whenever $(x_1, x_2, x_3 \ldots)$ and $(y_1, y_2, y_3, \ldots)$ are both elements of M and there is an integer n such that $x_n = y_n$, then $(x_1, x_2, \ldots, x_n, y_{n+1}, y_{n+2}, \ldots)$ is also an element of M. Show that if a compact subset M of the Hilbert cube has the crossover property and if and only if there is a collection of closed closed sets $G_1, G_2, G_3, \ldots$ in $[0,1] \times [0,1]$ such that $M = \bigstar_{i=1}^{\infty} G_i$.

E3 Show that if a compact subset M of the Hilbert cube has the crossover property and $\sigma(M) = M$ if and only if there is a single closed subset G of $[0,1] \times [0,1]$ such that $M = \bigstar_{i=1}^{\infty} G$

E4 Find a proof of the Mountain Climbing Theorem online, and read about other applications.

E5 Show that $\bigstar_{i=1}^{\infty}$ is homeomorphic to an inverse limit with continuous single valued bonding maps between the spaces $G, G\bigstar G, G\bigstar G\bigstar G, \ldots$.

E6 Show that if G is a continuum in $[0,1] \times [0,1]$ such that $\pi_1(G) = \pi_2(G) = [0,1]$, and G contains the graph of a continuous function defined on all of $[0,1]$, then for each n, $\bigstar_{i=1}^{n} G$ is homeomorphic to a subcontinuum of $\bigstar_{i=1}^{\infty} G$.