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Continuum theory workshop
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Definition. Let $f: X \to \mathbb{R}$ be a continuous function from a metric space X into reals, and let $p \in X$. We say that f is *upper semi-open* at *p* if for every open set *U* containing *p* there is $\varepsilon > 0$ such that $[f(p), f(p)+\varepsilon) \cap f(X) \subseteq f(U)$.

Definition. Let $f: X \to \mathbb{R}$ be a continuous function from a metric space X into reals, and let $p \in X$. We say that f is *lower semi-open* at *p* if for every open set *U* containing *p* there is $\varepsilon > 0$ such that $(f(p)-\varepsilon, f(p)] \cap f(X) \subseteq f(U)$.

Definition. Let $f: X \to \mathbb{R}$ be a continuous function from a metric space X into reals, and let $p \in X$. We say that f is *open* at *p* if it is both upper and lower semi-open at *p*. We say that *f* is *open (upper semi-open, lower semi-open, respectively)* if it is such at every point.

Problem 1. Give an example of a function $f:[0,1] \to [0,1]$ that is:

a) open, but not a homeomorphism;
b) upper semi-open, but not open;
c) lower semi-open, but not open;
d) neither upper nor lower semi-open.

Problem 2. Let $f, g:[0,1] \to [0,1]$. Which of the following statements are true? Prove or find a counterexample.

a) If *f* and *g* are open, then $g \circ f$ is open.
b) If *f* and *g* are upper semi-open, then $g \circ f$ is upper semi-open.
c) If *f* is upper semi-open and *g* is open, then $g \circ f$ is upper semi-open.
d) If *f* is open and *g* is upper semi-open, then $g \circ f$ is upper semi-open.

Problem 3. (Some hyperspace knowledge required). Prove that any Whitney map $\omega: 2^X \to [0, \omega(X)]$ is upper semi-open.

Problem 4. (Some hyperspace knowledge required). Prove that any Whitney map $\omega: C(X) \to [0, \omega(X)]$ is open.

Definition. For a metric continuum *X* and a closed subset *A* of *X*, define $\operatorname{diam}(A) = \sup\{d(x,y) : x, y \in A\}$. Thus diam is a function from 2^X to $[0, \operatorname{diam}(X)]$.

Problem 5. Construct a metric on a continuum such that diam is an open fuction.

Problem 6. Construct a metric on a continuum such that diam is upper semi-open, but not open.

Problem 7. Construct a metric on a continuum such that diam is lower semi-open, but not open.

Problem 8. Construct a metric on a continuum such that diam is neither lower nor upper semi-open.

Problem 9. Prove that for every nondegenerate continuum there is a metric such that the diameter function is not upper semi-open.

Problem 10. Prove that for every nondegenerate continuum there is a metric such that the diameter function is not lower semi-open.

Problem 11. Is there a metric on the simple closed curve such that the diameter is an open function?

For the following problems I do not know answers.

Problem 12. Is it true that for any locally connected continuum X there is a metric on X for which the diameter is open?

Problem 13. Is it true that for any dendrite X there is a metric on X for which the diameter is open?