# Reconfiguration control strategy using Takagi–Sugeno model predictive control for network control systems – a magnetic levitation case study

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**Abstract:** This paper presents a reconfiguration control strategy for network control systems that makes use of a fuzzy Takagi–Sugeno model for predictive control. The dynamic behaviour of a network control system is modelled by using a real-time implementation of the scheduling algorithm. Here, this is applied for a magnetic levitation system, as a plant that is also modelled using a fuzzy Takagi–Sugeno approach. Thus this paper covers several design issues, such as how to model a computer network, a plant, and a reconfiguration control strategy, as well as how the reconfiguration control strategy is modified using the fuzzy approach.

**Keywords:** reconfiguration control strategy, network control systems, fuzzy Takagi–Sugeno, model predictive control, real-time, magnetic levitation system, fault isolation, time delay modification

## 1 INTRODUCTION

Reconfiguration is a transition that modifies the structure of a system so it changes its representation of states. Here, it is used as a feasible approach for fault isolation, and it is also a response to time delay modification. In control systems, several modelling strategies for managing time delay within control laws have been studied by different research groups. Nilsson [**1**] proposes the use of a time delay scheme integrated to a reconfigurable control strategy, based on a stochastic methodology; Jiang and Zhao [**2**] describe how time delays are used as uncertainties, which modify pole placement of a robust control law; Izadi and Blanke [**3**] present an interesting case of a fault-tolerant control approach related to time delay coupling; Lian *et al.* [**4**] present a modelling of the dynamic system considering time delays due to communication networks (which is the basis for the present approach); Blanke *et al.* [**5**] study reconfigurable control from the point of view of structural modification, establishing a logical relation between dynamic variables and the respective faults; Thompson [**6**] and Benítez-Pérez and Garcia-Nocetti [**7**] consider that reconfigurable control strategies perform a combined modification of system structure and dynamic response, and thus this approach has the advantage of bounded modifications over system response; and finally, Almeida *et al.* [**8**] present a scheduling management for real-time distributed systems which aids the understanding of the time delay due to factors external to the distributed system, such as faults.

Normally, when a fault occurs during the operation of a system, a respective fault tolerance strategy is applied. However, applying such a fault tolerance strategy is not enough to maintain the performance of the system, since dynamic conditions are modified. Therefore, it seems to be necessary to take into account current conditions in order to maintain system performance, even degraded performance. Thus, this paper proposes a novel technique based on fuzzy model predictive control (MPC), and considering bounded variable time delays. Here, local faults and inherent time delays are used as necessary conditions for control design.

The approach adopted here makes use of a case study that takes time delays due to communication

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as deterministic measured variables: a light sensor, for which a fault is established as a deviation bigger than 50 per cent of the current value, and the related time delays are used for modelling the control law. For this, a fuzzy MPC law [**9**] is used, where time delays result from the deterministic reconfiguration of communications due to a scheduling algorithm. MPC is used for managing extended horizons from system inputs and outputs, to determine several scenarios modified by time delays. Recent results encourage this approximation, as shown in references [**10**] and [**11**].

For experimental purposes, the following considerations are taken into account.

1. Faults are strictly local in peripheral elements, and these are tackled by simply eliminating the faulty element. In fact, faults are considered as catastrophic and local.
2. Time delays are bounded and restrictive to the scheduling algorithms.
3. Global stability is reached by using a classical control strategy.
4. For each fault scenario, the combination tends to be globally stable, although a fuzzy Takagi–Sugeno (TKS) is used.

The objective of this paper is to present a reconfiguration control strategy developed from the time delay knowledge, as well as local fault effects within a distributed system environment, for a magnetic levitation case study. The novelty is to propose a TKS MPC for a network control system (NCS) based on the defined reconfiguration.

## 2 STRUCTURAL RECONFIGURATION ALGORITHM

This paper focuses on the reconfiguration of the control law due to local faults and consequent time delays, as shown in Fig. 1. Time delays are measurable and bounded, according to a real-time scheduling algorithm. Here, the scheduling algorithm used is the 'earliest deadline first' (EDF) algorithm [**12**]. 1

From Fig. 1, it is noticeable that structural reconfiguration takes place as a result of EDF, which makes use of an ART (adaptive resonance theory)-2A 2 neural network [**13**], whose action causes a control law transition. The aim here is to study how this transition is carried out when using a fuzzy TKS approach [**9**] based on MPC.

The core of the structural reconfiguration algorithm is to perform an on-line reconfiguration by using an ART-2A neural network [**14**] to classify valid and non-valid plans.

1. First, the ART-2A is trained off-line, using valid and non-valid plans from EDF evaluation and case study response.
2. Based on this training procedure, two main types of reconfigurations are determined: suitable reconfigurations and non-trustable reconfigurations.
3. During the on-line stage, ART-2A allows classification from new plans.
4. If the response of ART-2A represents a valid plan, the reconfiguration is performed; otherwise the proposed plan is rejected.

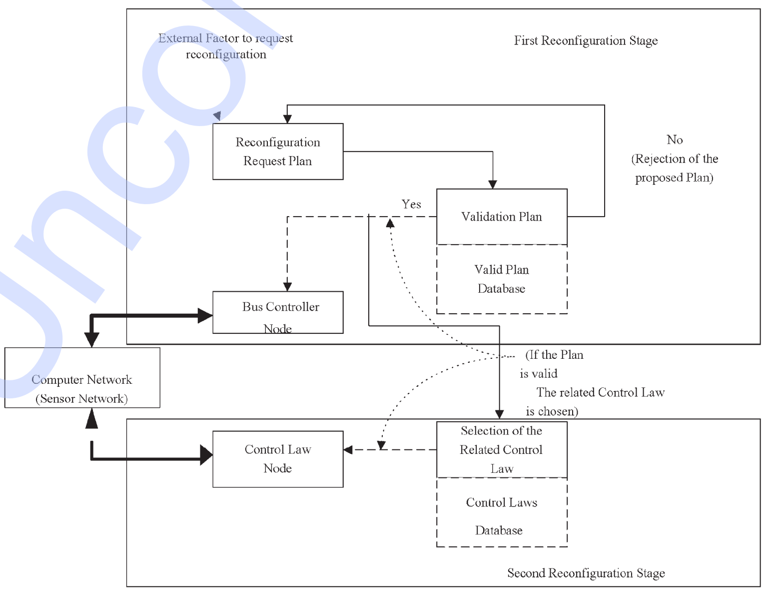


**Fig. 1** General structure of a reconfigurable system over a computer network

5. An important constraint is that ART-2A cannot learn new plans during the on-line stage, as a safety precaution.

For a NCS, the communication network strongly affects the dynamics of the system, expressed as a time variance that exposes a non-linear behaviour. Such non-linearity is addressed by incorporating time delays. From real-time system theory, it is known that time delays are bounded even in the case of causal modifications due to external effects. Using this representation, time delays are counted using simple addition, as described in the Section 3.

## 3 CASE STUDY AND RECONFIGURATION APPROACH

The case study here is a magnetic system integrated to a computer network as shown in Fig. 2 [**15**]. The dynamics of this case study are expressed in terms of a transfer function as

$$G_{\mathrm{bi}}(s) = \frac{-k_{\mathrm{bdc}} w_{\mathrm{b}}^2}{s^2 - w_{\mathrm{b}}^2} \tag{1}$$

$$k_{\mathrm{bdc}} = \frac{x_{\mathrm{bo}}}{I_{\mathrm{co}}}$$
$$w_{\mathrm{b}} = \sqrt{\frac{2g}{x_{\mathrm{bo}}}} \tag{2}$$

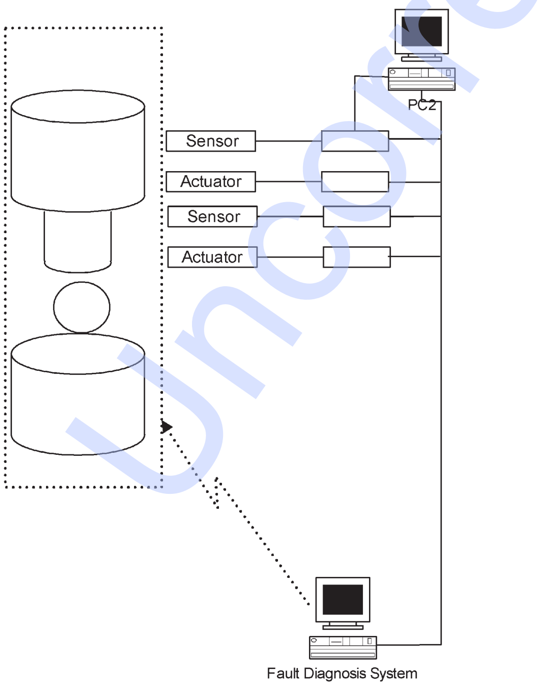


**Fig. 2** Magnetic levitator case study

where

$g$ is the force of gravity;
$I_{\mathrm{co}}$ is the current in the coil;
$x_{\mathrm{bo}}$ is the distance from coil to the ball position.

For experimental purposes, Fig. 3 presents a time diagram for a fault-free scenario, while Fig. 4 presents a time diagram for a fault scenario considering fault masking. In both these figures, $s_1$ and $s_2$ represent optic sensor nodes (in this case two light sensors are used), C is the control node, and $A_1$ is the actuator node. When a fault occurs, EDF is used, and the ART-2A re-organizes task execution according to time restrictions. Notice that in Fig. 3 and Fig. 4 the maximum time delays are bounded.

Both scenarios are local with respect to the magnetic levitation system: they are not periodic, although nodes are periodic. As both scenarios are bounded, the consumption times are expressed as equation (3) and equation (4) (from Fig. 3 and Fig. 4, respectively). For a fault-free scenario, time delay is

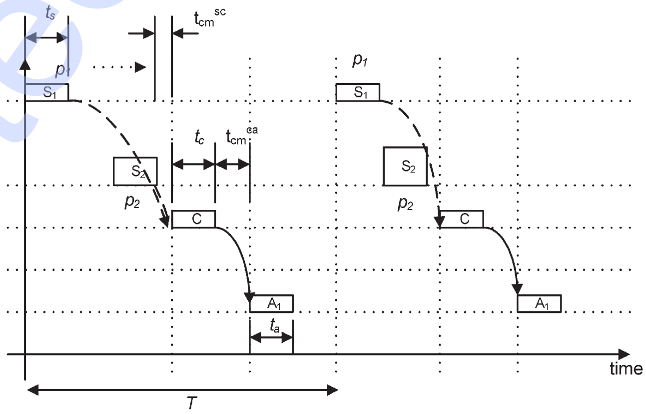


**Fig. 3** Fault-free scenario

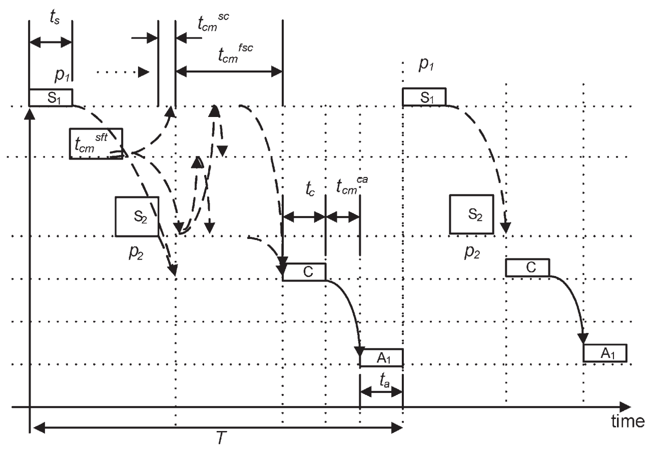


**Fig. 4** Fault scenario considering fault masking

expressed as

$$T = 3t_s + t_{cm}^{sc} + t_c + t_{cm}^{ca} + t_a \quad (3)$$

while for a fault scenario, time delay is expressed as

$$T = 3t_s + t_{cm}^{sc} + t_{cm}^{sft} + t_{cm}^{fsc} + t_c + t_{cm}^{ca} + t_a \quad (4)$$

where

$t_s$ is the time consumed by sensors;
$t_{cm}^{sc}$ is the communication time between sensor and control;
$t_{cm}^{sft}$ is the communication time between sensor and fault tolerance module;
$p_1$ and $p_2$ are consumption delays;
$t_{cm}^{fsc}$ is the time consumed for the fault sensor to send messages to its neighbour and produce agreement;
$t_c$ is the time consumed by the control node;
$t_{cm}^{ca}$ is the communication time between controller and actuator;
$t_a$ is the time consumed by the actuator.

Time values of these delays are averaging among components. From both scenarios, a *fault tolerance* element is defined as one that represents some extra communication for control performance, although it masks any local fault from sensors. From this time boundary, in both scenarios, it is feasible to implement some control strategies. Considering this, two possible fault cases are feasible:

(a) one local fault;
(b) several local faults.

From these two possible fault cases, the latter is a worst-case scenario, related to several local faults that have an impact on the global control strategy. However, the first case presents a minor degradation on the global control strategy. Despite this degradation, the system is expected to keep its functionality given the fault tolerance strategy and the local time delays integrated into related controllers. Faults are local failures of light capture on each optic sensor (three sensors are used). The fault-tolerant strategy is performed within the fault-tolerance element. Local faults are related to local values' deviations from current measures. Moreover, such local faults are neither catastrophic, nor permanent.

Considering the two possible fault cases, Table 1 contains the measured values for local and global time delays. As the time delays are bounded, the plant model is defined as an integration of the original plant and the control, i.e. from an integration of equation (1) and equation (2) (Fig. 5).

The plant model has the following dynamics

$$x(k+1) = a^p x(k) + B^p u(k)$$
$$y = c^p x(k) \quad (5)$$

where:

$a^p \in \Re^{n \times n}$;
$c^p \in \Re^{1 \times n}$;
$B^p \in \Re^{n \times 1}$ are matrices related to the plant;
$x(k)$, $u(k)$, and $y(k)$ are states, inputs, and outputs, respectively.

In particular, $B^p$ is defined as:

$$B^p = \sum_{i=1}^{N} \rho_i B_i \sum_{j=1}^{M} \int e^{-a^p(t-\tau)} d\tau \quad (6)$$

where

$\rho_i = \{0, 1\}$, $\sum_{i=1}^{N} \rho_i = 1$;

**Table 1** Time delays related to local communications

| | | |
|---|---|---|
| Configuration 1 | Local time delays | 1 ms |
| Several local faults | Global time delays | 5 ms |
| Configuration 2 | Local time delays | 1 ms |
| One local faults | Global time delays | 3 ms |

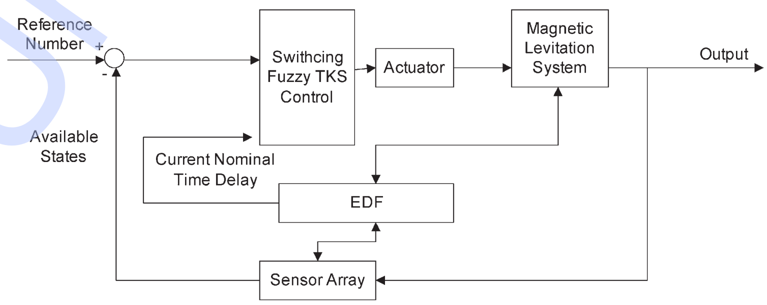


**Fig. 5** Plant and control integration

$N$ is the total number of possible faults;
$M$ is the involved time delays from each fault;
$\tau^i_{j-1}$ and $\tau^i_j$ are current communication time delay, $\sum_{j=1}^{M}\tau^i_j \leqslant T$ where $T$ is the total transport delay of the cycle and depends on the faults scenarios.

Thus, $B_i$ is an array

$$B_i = \begin{bmatrix} b_1 \\ b_2 \\ 0_i \\ \vdots \end{bmatrix}$$

where

$b_1 \rightarrow b_N$ are the elements conformed at the input of the plant (such as actuators);
$0_i$ is the lost element due to local actuator fault.

$B^p$ represents only one scenario (equation (6)). A further definition of a current $B^p_i$ considers local actuator faults and related time delays:

$$B^p_i = B_i \sum_{j=1}^{M} \int_{\tau^i_j}^{\tau^i_{j-1}} \mathrm{e}^{-a^p(t-\tau)}\mathrm{d}\tau \tag{7}$$

For simplicity, $B^p_i$ is used in order to describe local linear plants.

From this representation, a fuzzy plant is defined as follows, taking into consideration each time delay, fault cases, and the related fuzzy rules

$$\begin{aligned} r_i : &\text{ if } x_1 \text{ is } \mu_{1i} \text{ and } x_2 \text{ is } \mu_{2i} \text{ and} \ldots \text{and } x_l \text{ is } \mu_{li} \\ &\text{then } a^p_i x(k) + B^p_i u(k) \end{aligned} \tag{8}$$

where

$\{x_1 x_2 \cdots x_l\}$ are current state measures;
$l$ is the number of states;
$i = \{1, \ldots, N\}$ is one of the fuzzy rules;
$N$ is the number of the rules, which is equal to the number of possible faults;
$\mu_{ij}$ are the related membership functions, which are Gaussian defined as

$$\mu_{ij}(y_\alpha, u_\alpha) = \exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_\alpha - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right] \tag{9}$$

where $c_{ij}$ and $\sigma_{ij}$ are parameters to be tuned.

The representation of the plant as an integrated system with the control is thus based on the centre of area de-fuzzification method [**16**]. From this representation of a global non-linear system, it is necessary to define a global stability condition as a result of this fuzzy system. This is given considering the fuzzy logic control approach. The result allows the integration of the non-linear stages and transitions basically to a group of linear plants. From the point of view of the approach, taking the input of the plant as consequent, this is defined as fuzzy MPC as follows

$$u_i = (S^T_i \bar{Q} S_i + \bar{R})^{-1} S_i \bar{Q}(w - p_i) \tag{10}$$

where

$p_i$ is a recursive variable that is tuned according to plant timing horizons and inherent time delays (defined in equation (12));
$w$ are the future set points;
$u_i$ is the control output;
$\mathbf{Q}$ and $\mathbf{R}$ are positive definite weight matrices defined as

$$\bar{Q} = \mathrm{Diag}(Q)$$

$$\bar{R} = \mathrm{Diag}(R)$$

$\mathbf{S}$ represents the effect of future outputs and from the integration to antecedent representation of the fuzzy logic system (equation (8)), over $N_\mathrm{p}$ and $N_\mathrm{c}$ horizons defined by the user

$$\mathbf{S} = \begin{bmatrix} S_{N_{p1}} & S_{N_{p1}+1} & \cdots & 0 \\ S_{N_{p2}-1} & S_{N_{p1}} & S_{N_{p1}+1} & \\ \vdots & \vdots & \ddots & \\ S_{N_{p2}} & S_{N_{p2}-1} & \cdots & S_{N_{p2}-N_c} \end{bmatrix} \tag{11}$$

in which

$$\begin{aligned} s_j &= y_0 \quad \forall j \leqslant n_\mathrm{d} \\ s_j &= \sum_{i=1}^{Na} a_i S_{j-i} + \sum_{i=1}^{Nb} B^{p_i} \quad j > n_\mathrm{d} \\ p_i &= \sum_{j=1}^{Na} a_j p_{i-j} + \sum_{j=1}^{Nb} b_j B^p_j u(k - j - n_\mathrm{d} + i) + c \end{aligned} \tag{12}$$

Figure 6 shows how these horizons take place in time.


**Fig. 6** Time horizons with respect to time delays, horizon samples, and $k$ sampling time

3 In Fig. 6, $N_a$ and $N_b$ are the horizon samples, $k$ is the sampling time, $l$ is the related time delay within the sampling time, $n_d$ is the minimum discrete deadtime. In equation (12), the parameters of the plant are presented as $a_j$ and $B_i^p$ where

$$b_j = f\left(B_j^p\right)$$

and

$$B_i^p = \int_{t_i}^{t_{i+1}} e^{(a^p - \tau)} d\tau B$$

From the integration to antecedent representation of the fuzzy system (equation (8))

$$D_i(y_\alpha, u_{\alpha'}) = \prod_{j=1}^{N_p} \mu_{ij}(y_\alpha, u_{\alpha'}) = \prod_{j=1}^{N_p} \exp\left[-\left(\frac{y_\alpha - c_{ij}^y}{\sigma_{ij}^y}\right)^2 - \left(\frac{u_{\alpha'} - c_{ij}^u}{\sigma_{ij}^u}\right)^2\right] \tag{13}$$

$$\alpha = \begin{cases} 0 & j > N_p \\ j & 1 \leqslant j \leqslant N_p \end{cases} \qquad \alpha' = \begin{cases} 0 & 1 \leqslant j \leqslant N_p \\ j & j > N_p \end{cases}$$

where

$N_p$ is the number of possible inputs for the fuzzy plant;
$y$ is the output of the plant;
$u$ is the plant input.

For the antecedent part of fuzzy control $\Omega_i$

$$\Omega_i(y_\alpha, u_{\alpha'}) = \prod_{j=1}^{N_A} \mu_{ij}(y_\alpha, u_{\alpha'}) = \prod_{j=1}^{N_A} \exp\left[-\left(\frac{y_\alpha - c_{ij}^y}{\sigma_{ij}^y}\right)^2 - \left(\frac{u_{\alpha'} - c_{ij}^u}{\sigma_{ij}^u}\right)^2\right] \tag{14}$$

$$\alpha \begin{cases} 0 & j > N_A \\ j & 1 \leqslant j \leqslant N_A \end{cases} \qquad \alpha' \begin{cases} 0 & 1 \leqslant j \leqslant N_A \\ j & j > N_A \end{cases}$$

where $N_A$ is the number of possible inputs for the fuzzy controller, following that expressed in Fig. 6.

In this case, fault conditions are presented as the results of local time delays more than the actual loss of current measure. Remember that equation (3) and equation (4) are the basis for time delay by using EDF as the scheduling algorithm, presented as

$$3t_s + t_{cm}^{sc} + t_c + t_{cm}^{ca} + t_a < T$$

Thus, the plant representation is given by equation (15) considering time delays

$$x(k+1) = \frac{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})\left[a_i x(k) + B_i^p u(k)\right]}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})} \tag{15}$$

where $N$ is the number of rules, and the plant input is defined as considering time delays expressed in equation (14)

$$u(k) = \frac{\sum_{i=1}^{N} \Omega_i(y_\alpha, u_{\alpha'})\left(S_k^T \bar{Q} S_k + \bar{R}\right)^{-1} S_k \bar{Q}(w - p_k)}{\sum_{i=1}^{N} \Omega_i(y_\alpha, u_{\alpha'})} \tag{16}$$

Substituting in equation (15)

$$x(k+1) = \frac{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})\left(a_i x(k) + B_i^p \frac{\sum_{i=1}^{N} \Omega_i(y_\alpha, u_{\alpha'})\left(S_k^T \bar{Q} S_k + \bar{R}\right)^{-1} S_k \bar{Q}(w - p_k)}{\sum_{i=1}^{N} \Omega_i(y_\alpha, u_{\alpha'})}\right)}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})} \tag{17}$$

On the other hand, in order to establish valid horizons considering time delays and failures, an MPC strategy is used, and therefore, the cost function in MPC is defined as

$$J=\sum_{i=1}^{N_p} B_i^p(\mathrm{ref}_i-y_i)^2+\sum_{i=1}^{N_A}\delta_i(u_i)^2 \tag{18}$$

where $\mathrm{ref}_i$ and $y_i$ are the reference and output values respectively. This equation can be rewritten as

$$J=\sum_{k=1}^{N_p} B_k^p[\mathrm{ref}_k-Cx(k-1)]^2+\sum_{k=1}^{N_A}\delta_k(u_k)^2 \tag{19}$$

Considering the variables $x$ and $u_i$ defined in equation (15) and equation (16)

$$J=\sum_{k=1}^{N_p} B_k^p\left(\mathrm{ref}_k-C\frac{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})\left(a_i x(k-2)+B_i^p u(k-2)\right)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right)^2+\sum_{k=1}^{N_A}\delta_k\left(\frac{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})\left(S_k^T\bar{Q}S_k+\bar{R}\right)^{-1}S_k\bar{Q}(w-p_k)}{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})}\right) \tag{20}$$

Since the values of **Q**, **S**, and **R** are defined as positive definite matrices in equation (10), it is necessary to obtain the partial derivatives for each variable in order to get the optimal values as:

$$\frac{\partial J}{\partial B_k^p}=2\sum_{k=1}^{N_p} B_k^p[\mathrm{ref}_k-Cx(k-1)] \tag{21}$$

$$\frac{\partial J}{\partial \delta_k}=2\sum_{k=1}^{N_A}\delta_k(u_k) \tag{22}$$

$$\frac{\partial J}{\partial a_i}=2C\sum_{k=1}^{N_p} B_k^p\left\{\mathrm{ref}_k-C\frac{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})\left[a_i x(k-2)+B_i^p u(k-2)\right]}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right\}\left[\frac{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})x(k-2)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right] \tag{23}$$

$$\frac{\partial J}{\partial B_i^p}=2\sum_{k=1}^{N_p} B_k^p\left\{\mathrm{ref}_k-C\frac{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})\left[a_i x(k-2)+B_i^p u(k-2)\right]}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right\}\left[\frac{Cu(k-2)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right] \tag{24}$$

$$\frac{\partial J}{\partial D_i}=2C\sum_{k=1}^{N_p} B_k^p[\mathrm{ref}_k-Cx(k-1)]\left\{\frac{\sum_{i=1}^{N}\left[a_i x(k-2)+B_i^p u(k-2)\right]-Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right\} \tag{25}$$

$$\frac{\partial J}{\partial \Omega_i}=2\sum_{k=1}^{N_A}\delta_k u(k)\left[\frac{\sum_{i=1}^{N}\left(S_k^T\bar{Q}S_k+\bar{R}\right)^{-1}S_k\bar{Q}(w-p_k)-Nu(k)}{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})}\right] \tag{26}$$

Using equation (13) to obtain the partial derivatives of $D_i$ with respect to $c_{ij}$ and $\sigma_{ij}$

$$\frac{\partial D_i}{\partial c^y_{ij}} = -2\sum_{k=1}^{N_p}\left[\frac{y_\alpha - c^y_{ik}}{(\sigma^y_{ik})^2}\right]\exp\left[-\left(\frac{y_\alpha - c^y_{ik}}{\sigma^y_{ik}}\right)^2\right]\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right] \tag{27}$$

$$\frac{\partial D_i}{\partial c^u_{ij}} = -2\sum_{k=1}^{N_p}\left[\frac{u_{\alpha'} - c^u_{ik}}{(\sigma^u_{ik})^2}\right]\exp\left[-\left(\frac{u_{\alpha'} - c^u_{ik}}{\sigma^u_{ik}}\right)^2\right]\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right] \tag{28}$$

Analogously, for $\sigma^y_{ij}$ and $\sigma^u_{ij}$

$$\frac{\partial D_i}{\partial \sigma^y_{ij}} = -\frac{1}{2}\sum_{k=1}^{N_p}\frac{(y_\alpha - c^y_{ik})^2}{(\sigma^y_{ik})^3}\exp\left[-\left(\frac{y_\alpha - c^y_{ik}}{\sigma^y_{ik}}\right)^2\right]\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right] \tag{29}$$

$$\frac{\partial D_i}{\partial \sigma^u_{ij}} = -\frac{1}{2}\sum_{k=1}^{N_p}\frac{(u_{\alpha'} - c^u_{ik})^2}{(\sigma^u_{ik})^3}\exp\left[-\left(\frac{u_{\alpha'} - c^u_{ik}}{\sigma^u_{ik}}\right)^2\right]\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right] \tag{30}$$

Applying the previous results, and using equation (25)

$$\begin{aligned}\frac{\partial J}{\partial c^y_{ij}} = \frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial c^y_{ij}} = 2C\sum_{k=1}^{N_p}B^p_k[\mathrm{ref}_k - Cx(k-1)]&\left\{\frac{\sum_{i=1}^{N}[a_i x(k-2) + B^p_i u(k-2)] - Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})}\right\}\\ \times\left\{-2\sum_{k=1}^{N_p}\left[\frac{y_\alpha - c^y_{ik}}{(\sigma^y_{ik})^2}\right]\exp\left[-\left(\frac{y_\alpha - c^y_{ik}}{\sigma^y_{ik}}\right)^2\right]\right.&\left.\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right]\right\}\end{aligned} \tag{31}$$

and applying the same for $c^u_{ij}$

$$\begin{aligned}\frac{\partial J}{\partial c^u_{ij}} = \frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial c^u_{ij}} = 2C\sum_{k=1}^{N_p}B^p_k[\mathrm{ref}_k - Cx(k-1)]&\left\{\frac{\sum_{i=1}^{N}[a_i x(k-2) + B^p_i u(k-2)] - Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})}\right\}\\ \times\left\{-2\sum_{k=1}^{N_p}\left[\frac{u_{\alpha'} - c^u_{ik}}{(\sigma^u_{ik})^2}\right]\exp\left[-\left(\frac{u_{\alpha'} - c^u_{ik}}{\sigma^u_{ik}}\right)^2\right]\right.&\left.\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right]\right\}\end{aligned} \tag{32}$$

Analogously for $\sigma^y_{ij}$ and $\sigma^u_{ij}$

$$\begin{aligned}\frac{\partial J}{\partial \sigma^y_{ij}} = \frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial \sigma^y_{ij}} = 2C\sum_{k=1}^{N_p}B^p_k[\mathrm{ref}_k - Cx(k-1)]&\left\{\frac{\sum_{i=1}^{N}[a_i x(k-2) + B^p_i u(k-2)] - Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})}\right\}\\ \times\left\{-\frac{1}{2}\sum_{k=1}^{N_p}\frac{(y_\alpha - c^y_{ik})^2}{(\sigma^y_{ik})^3}\exp\left[-\left(\frac{y_\alpha - c^y_{ik}}{\sigma^y_{ik}}\right)^2\right]\right.&\left.\prod_{j=1,\,j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c^y_{ij}}{\sigma^y_{ij}}\right)^2 - \left(\frac{u_{\alpha'} - c^u_{ij}}{\sigma^u_{ij}}\right)^2\right]\right\}\end{aligned} \tag{33}$$

$$\frac{\partial J}{\partial \sigma_{ij}^{u}} = \frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial \sigma_{ij}^{u}} = 2C\sum_{k=1}^{N_p} B_k^p[\mathrm{ref}_k - Cx(k-1)]\left\{\frac{\sum_{i=1}^{N}\left[a_i x(k-2) + B_i^p u(k-2)\right] - Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha, u_{\alpha'})}\right\}$$

$$\times\left\{-\frac{1}{2}\sum_{k=1}^{N_p}\frac{(u_{\alpha'} - c_{ik}^{u})^2}{(\sigma_{ik}^{u})^3}\exp\left[-\left(\frac{u_{\alpha'} - c_{ik}^{u}}{\sigma_{ik}^{u}}\right)^2\right]\prod_{j=1,\, j\neq k}^{N_p}\exp\left[-\left(\frac{y_\alpha - c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^2 - \left(\frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^2\right]\right\} \tag{34}$$

A similar procedure is used to obtain the partial derivatives with respect to $c_{ij}$ and $\sigma_{ij}$, using equation (14). The optimization procedures of $\delta$, $Q$, and $R$ are left to the use of this multivariable optimization procedure, since these are design variables.

## 4 RESULTS

Using this implementation, several experiments are carried out, the results of which are presented in terms of fault presence and its related fault tolerance strategy to overcome system lack of performance regarding a horizon. How the system responds to the fault tolerance strategy is presented as follows, showing the error response from different separation values between membership functions, according to Table 2. The experimental set-up consists of two light sensors connected through two microcontrollers and CANBUS, one light sensor connected to a Q4 analogue-to-digital (A/D) card, one actuator connected to a 612 A/D card; each of the elements has a consumption delay of 1 ms.

Figure 7 shows fault-free scenarios for 10 per cent and 15 per cent of this separation between membership functions, as presented in reference [**11**]. Faults refer to light capture that detects current movement of the ball of the magnetic levitation case study. Both faults are local deviations of current responses from light sensors. These are not catastrophic faults, but only partial deviation from current measurements. These faults are presented locally, and within a time frame bigger than the sampling time.

For both fault scenarios, the response of the systems is shown in Table 3. Figure 8 shows the error response for each fault scenario, when switching from one sensor to another, using the separation of

**Table 2** Fault-free scenario for different per cent of separation values between membership functions

| Separation (%) | Integral of the error |
|---|---|
| 10 | 0.4400 |
| 15 | 0.4495 |

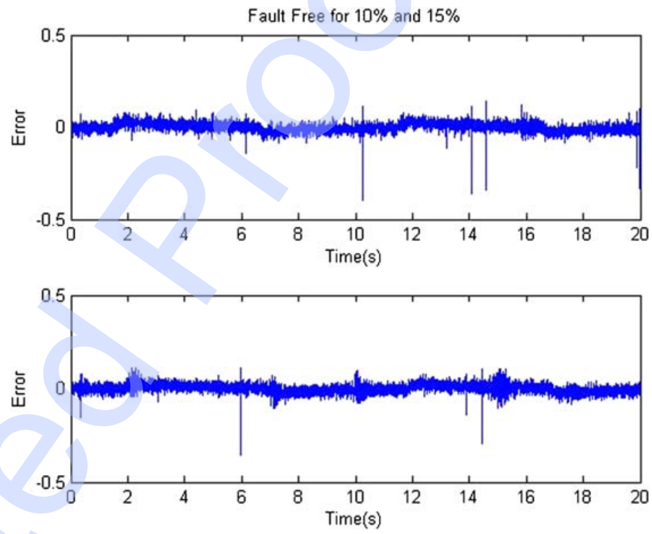


**Fig. 7** Error response from fault-free scenario

**Table 3** The integral of the error for the fault scenario

| Separation (%) | Integral of the error |
|---|---|
| 10 | 0.5003 |
| 15 | 0.4567 |

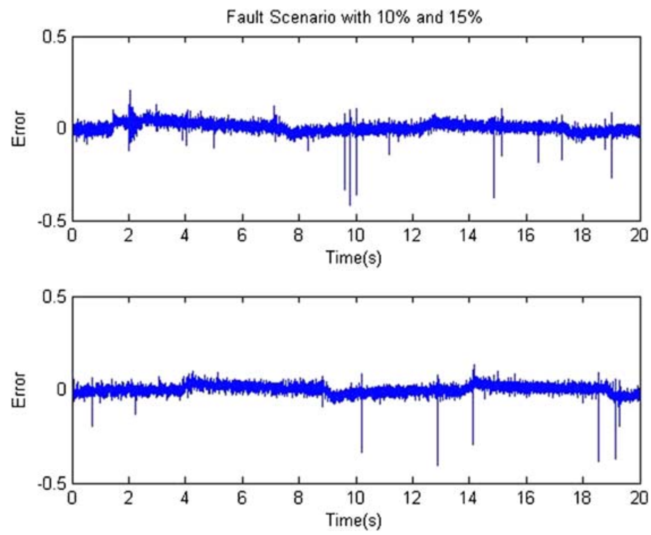


**Fig. 8** Error response from fault scenario 1

10 per cent and 15 per cent. Figure 9 shows current response from the error signal considering the first fault scenario. Alternatively, Fig. 10 presents the response of the system compared with the current set point.

This last example presents the reconfiguration control strategy based on the decision-maker module and the related fuzzy MPC. Active switching control is performed using a fuzzy TKS approach when a local fault appears, and fault-tolerant node is activated. Such a reconfiguration control strategy is feasible owing to the knowledge about time delays. Notice that the consumption time of the reconfiguration control strategy is neglected, since it is considered to be part of control performance. It is obvious that fault

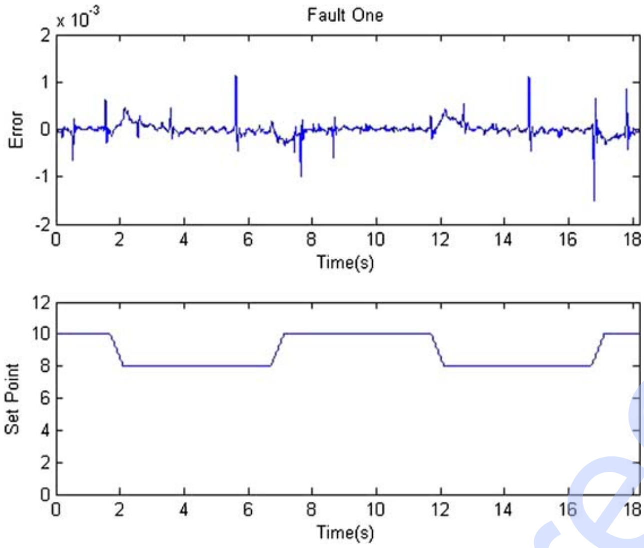


**Fig. 9** Error response for first fault scenario considering current set point

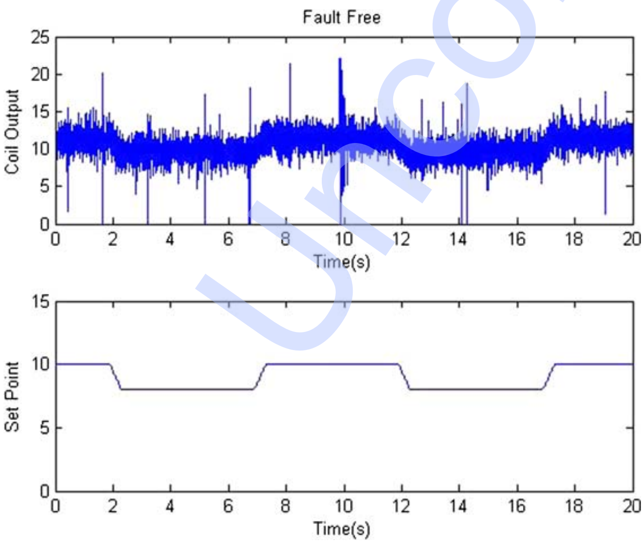


**Fig. 10** Set point and system response from case study

presence is measurable: if a local fault cannot be detected, the strategy becomes useless. Alternatively, local time delay management refers to the use of a quasi-dynamic scheduler to propose dynamic reconfiguration, based on current system behaviour rather than predefined scenarios. Error responses present noise conditions that are inherent to system response.

To define the communication network performance, the use of xPC Target is necessary. Such a network implementation uses message transactions that are implemented in the real-time workshop toolbox from MATLAB [**17**]. For the case study, two types of computer network are used: CANbus and Ethernet. Both networks present no further time delay difference because network size is kept quite small. Actual set-up of both computer networks is presented in Table 4.

## 5 CONCLUDING REMARKS

The present paper presents an approach for integrating two techniques in order to perform reconfiguration: structural reconfiguration and control reconfiguration. These two techniques are applied in cascade. Although there is no formal verification for following this sequence, it has been adopted because structural reconfiguration provides stable conditions for control reconfiguration. Moreover, the use of a real-time scheduling algorithm to approve or disapprove changes in the behaviour of a computer network allows bounding time delays during a specific time frame. This local time delay allows the design of a control law that is capable of coping with new conditions.

Preliminary results show that the proposed reconfiguration control strategy is feasible as long as the use of a wide enough horizon predetermines which control law is adequate. This is accomplished by the composition of two algorithms: one responsible for structural reconfiguration (and implemented here as ART-2A) and the second responsible for control reconfiguration (here based on fuzzy TKS and MPC). The importance of this approach is that control conditions are strictly bounded to a certain

**Table 4** Current conditions on computer network

| | CANBUS | ETHERNET |
|---|---|---|
| Bandwidth | 1 Mbits/s | 10 Mbits/s |
| Number of nodes | 4 | 3 |
| Bus loading | 64 Bytes/node (there are no more nodes transmitting on this network) | 1 KByte/node (there are no more nodes transmitting on this network) |

response. Future work will be related to integrating dynamic scheduling algorithms and a formal stability probe for this implementation.

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## REFERENCES

1 **Nilsson, J.** *Real-time control with delays*. PhD Thesis, Department of Automatic Control, Lund Institute of Technology, Sweden, 1998.

2 **Jiang, J.** and **Zhao, Q.** Reconfigurable control based on imprecise fault identification. In Proceedings of the American Control Conference, IEEE, San Diego, June 1999, pp. 114–118.

3 **Izadi-Zamanabadi, R.** and **Blanke, M.** A ship propulsion system as a benchmark for fault-tolerant control. *Control Engng Practice*, 1999, **7**, 227–239.

4 **Lian, F., Moyne, J.,** and **Tilbury, D.** Network design consideration for distributed control systems. *IEEE Trans., Control Systems Technol.*, 2002, **10**(2), 297–307.

5 **Blanke, M., Kinnaert, M., Lunze, J.,** and **Staroswiecki, M.** *Diagnosis and fault tolerant control*, 2003 (Springer, Berlin).

6 **Thompson, H.** Wireless and internet communications technologies for monitoring and control. *Control Engng Practice*, 2004, **12**, 781–791.

7 **Benítez-Pérez, H.** and **García-Nocetti, F.** *Reconfigurable distributed control*, 2005 (Springer, Berlin).

8 **Almeida, L., Pedreiras, P.,** and **Fonseca, J. A.** The FTT-CAN protocol: why and how. *IEEE Trans., Ind. Electronics*, 2002, **49**(6), 1189–1201.

9 **Abonyi, J.** *Fuzzy model identification for control*, 2003 (Birkhäuser).

10 **Benítez-Pérez, H.** Real-time distributed control a diverse approach for nonlinear problem. *Nonlinear Analysis: Hybrid Systems and Applications*, 2008, **2**(2), 474–490. DOI: 10.1016/j.nahs.2006.06.004.

11 **Benítez-Pérez, H., Solano-Gonzalez, J., Cardenas-Flores, F.,** and **García-Nocetti, F.** Fault classification 4
for a class of time variable systems by using a group of three ART2A networks. *Int. J. Control Intell. Systems*, 2008, **36**(1). DOI: 10.2316/Journal.201.2008.1.201-1820.

12 **Liu, L.** *Real-time systems*, 2002 (Wiley, Chichester).

13 **Garcia-Zavala, A., García-Nocetti, D. F.,** and **Benítez-Pérez, H.** Alternative method based upon planning scheduler for on-line reconfiguration using system performance. In Proceedings of the 5th IEEE Symposium and School on *Advanced distributed systems, ISSADS*, Lecture Notes in Computer Science 3563, 2005, pp. 141–152, ISBN 3-540-28063-4.

14 **Frank, T., Kraiss, K. F.,** and **Kuhlen, T.** Comparative 5
analysis of Fuzzy ART and ART-2A network clustering performance. *IEEE Trans., Neural Networks*, 1998, **9**(3).

15 Quanser Industrial: *Control solutions*, 2003, available from http://www.quanser.com/english/html/solutions/fs_soln_software_wincon.html.

16 **Driankov, D., Hellendoorn, H.,** and **Reinfrank, M.** *An introduction to fuzzy logic control*, 1994 (Springer-Verlag, Berlin).

17 **Hanselman, D.** and **Littlefield, B.** *Mastering MATLAB*, 2002 (Prentice Hall, Englewood Cliffs, New Jersey).