CHAPTER 18 · JOHANNES KEPLER (1571-1630)

Out of the night that covers me,
Black as the pit from pole to pole
I thank whatever gods there be
For my unconquerable soul.
—W. E. Henley (in hospital, 1875)

"For there is a musick where ever there is a harmony, order, or proportion: and thus far we may maintain the musick of the sphears; for those well-ordered motions and regular paces, though they give no sound unto the ear, yet to the understanding they strike a note most full of harmony."

From Religio Medici
Sir Thomas Browne (1642)

Kepler, the young German to whom Tycho Brahe left his tables, was well worthy of this trust. He grew into one of the greatest scientists of the age—perhaps equalled in his own time only by Galileo and later outshone only by Newton. As Sir Oliver Lodge points out, Tycho and Kepler form a strange contrast: Tycho "rich, noble, vigorous, passionate, strong in mechanical ingenuity and experimental skill, but not above the average in theoretical power and mathematical skill"; and Kepler "poor, sickly, devoid of experimental gifts, and unfitted by nature for accurate observation, but strong almost beyond competition in speculative subtlety and innate mathematical perception."[1] Tycho's work was well supported by royalty, at one time magnificently endowed; Kepler's material life was largely one of poverty and misfortune. They had in common a profound interest in astronomy and a consuming determination in pursuing that interest.

Kepler was born in Germany, the eldest son of an army officer. He was a sickly child, delicate and subject to violent illnesses, and his life was often despaired of. The parents lost their income and were reduced to keeping a country tavern. Young Johannes was taken from school when he was nine and continued as a servant till he was twelve. Ultimately he returned to school and went on to the University where he graduated second in his class. Meanwhile, his father abandoned his home and returned to the army; and his mother quarreled with her relations, including her son, who was therefore glad to get away. At first he had no special interest in astronomy. At the University he heard the Copernican system expounded. He adopted it, defended it in a college debate, and even wrote an essay on one aspect of it. Yet his major interests at that time seem to have been in philosophy and religion, and he did not think much of astronomy. But then an astronomical lecturership fell vacant and Kepler, who was looking for work, was offered it. He accepted reluctantly, protesting, he said, that he was not thereby abandoning his claim "to be provided for in some more brilliant profession." In those days astronomy had little of the dignity which Kepler himself later helped to give it. However, he set to work to master the science he was to teach; and soon his learning and thinking led to more thinking and enjoyment. "He was a born speculator just as Mozart was a born musician"[1]; and he *had* to find the mathematical scheme underlying the planetary system. He had a restless inquisitive mind and was fascinated by puzzles concerning numbers and size.[2] Like Pythagoras, he "was convinced that God created the world in accordance with the principle of per-

[1] Sir Oliver Lodge, *Pioneers of Science.*

[2] Most of us have similar delights, though less intense. You have probably enjoyed working on series of numbers, given as a puzzle or an "intelligence test," trying to continue the series. Try to continue each of the following. If you enjoy puzzling over them (as well as succeeding) you are tasting something of Kepler's happiness.

(a) 1, 3, 5, 7, 9, 11, . . . How does this series probably go on?
(b) 1, 4, 9, 16, 25, . . . ?
(c) 5, 6, 7, 10, 11, 12, 15, 16, . . . ?
(d) 2, 3, 4, 6, 8, 12, 14, 18, . . . ?
(e) 4, 7, 12, 19, 28, . . . ?
(f) 1 7 3 6 5 5 7 4 9 . . . ?
(g) 0 1 8 8 1 1 0 2 4 1 5 6 2 5 . . . ?

[Note that in (f) and (g) you must also find where to put the commas.]

FIG. 18-1. ? LAW RELATING SIZES OF PLANETARY ORBITS ?

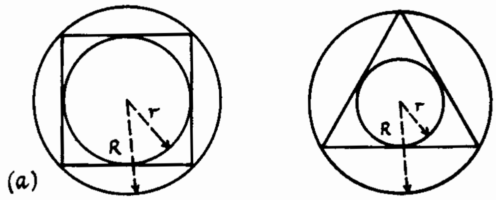


FIG. 18-1a. KEPLER'S FIRST GUESS
A regular plane figure (such as a square) can have a circle inscribed, to touch its sides. It can also have an outside circle, through its corners. Then that outside circle can be the inner circle for another, larger plane figure. The ratio of radii, R/r, is the same for all squares; and it has a different fixed value for all triangles. Geometrical puzzle: what is the fixed value of R/r for the inner and outer circles of a square? What is the value for a triangle?

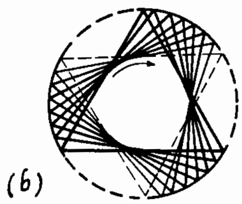


FIG. 18-1b. The same two circles can be generated by letting the figure (here a triangle) spin around its own center, in its own plane. Its corners will touch the outer circle, and its sides envelop the inner one.

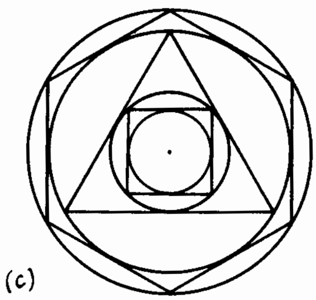


FIG. 18-1c. A series of regular plane figures, separated by inner and outer circles, provides a series of circles which might show the proportions of the planetary orbits. Even the best choice of figures failed to fit the solar system.

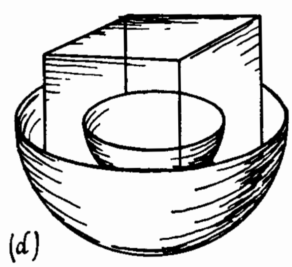


FIG. 18-1d. KEPLER'S SECOND GUESS
This shows the basis of Kepler's final scheme. He chose the order of regular solids that gave the best agreement with the known proportions of planetary orbits.

fect numbers, so that the underlying mathematical harmony . . . is the real and discoverable cause of the planetary motions."[3] Kepler himself said, "I brooded with the whole energy of my mind on this subject."

His mind burned with questions: Why are there only *six* planets? Why do their orbits have just the proportions and sizes they do? Are the times of the planets' "years" related to their orbit-sizes? The first question, "Why just six?" is characteristic of Kepler's times—nowadays we should just hunt for a seventh. But then there was a finality in facts and a magic in numbers. The Ptolemaic system counted seven planets (including Sun and Moon, excluding the Earth) and even had arguments to prove seven must be right.

Kepler tried again and again to find some simple relation connecting the radius of one orbit with the next. Here are *rough relative* radii from Tycho's observations, calculated for the Copernican scheme: 8 : 15 : 20 : 30 : 115 : 195. He tried to guess the secret in these proportions. Each guess meant a good deal of work, and each time he found it did not fit the facts he rejected that guess honestly. His mystical mind clung to the Greek tradition that circles are perfect; and at one time he thought he could construct a model of the orbits thus: draw a circle, inscribe an equilateral triangle in it, inscribe a circle in that triangle, then another triangle inside the inner circle, and so on. This scheme gives successive circles a definite ratio of radii, 2:1. He hoped the circles would fit the proportions of the planetary orbits if he used squares, hexagons, etc., instead of some of the triangles. No such arrangement fitted. Suddenly he cried out, "What have flat patterns to do with orbits in *space*? Use solid figures." He knew there are only five completely regular crystalline solid shapes (see Fig. 18-3). Greek mathematicians had proved there cannot be more than five. If he used these five solids to make the separating spaces between six spherical bowls, the bowls would define six orbits. Here was a wonderful reason for the number six. So he started with a sphere for the Earth's orbit, fitted a dodecahedron outside it with its faces touching the sphere, and another sphere outside the dodecahedron passing through its corners to give the orbit of Mars; outside that sphere he put a tetrahedron, then a sphere for Jupiter, then a cube, then a sphere for Saturn. Inside the Earth's sphere he placed two more solids separated by spheres, to give the orbits of Venus and Mercury.

[3] Sir William Dampier, *History of Science* (4th edn., Cambridge University Press, 1949).

THE REGULAR SOLIDS. A geometrical intelligence test

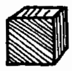
How many different shapes of regular solid are possible? To find out, follow argument (a); then try (b).

A regular solid is a geometrical solid with identical regular plane faces; that is, a solid that has:

all its edges the same length
all its face angles the same
all its corners the same
and all its faces the same shape.
(See opposite for shapes that do not meet the requirements.)

FIG. 18-2.

For example, a cube is a regular solid.



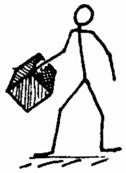
The faces of a regular solid might be:

all equilateral triangles
or all squares
or all regular pentagons
or . . . and so on . . .

(a) Here is the argument for square faces. Try to make a corner of a regular solid by having several corners of squares meeting there.

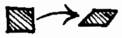
We already know that in a cube each corner has three square faces meeting there. Take three squares of cardboard and place them on the table like this, then try to pick up the place where three corners of squares meet. The squares will fold to make a cube corner.

Therefore we can make a regular solid with three square faces meeting at each of the solid's corners. (We need three more squares to make the rest of the faces and complete the cube.)



Could we make another regular solid, with only one, or two, or four square faces meeting at a corner?

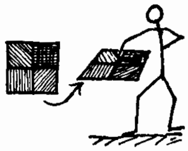
With *one square*, we cannot make a solid corner.



With *two squares*, we can only make a flat sandwich.

With *three squares*, we make a cubical corner, leading to a cube.

With *four squares* meeting at a corner, they make a flat sheet there, and cannot fold to make a corner for a closed solid.



Thus, SQUARES CAN MAKE ONLY *ONE* KIND OF REGULAR SOLID, A CUBE.

(b) Now try for yourself with regular pentagons, and ask how many regular solids can be made with such faces.

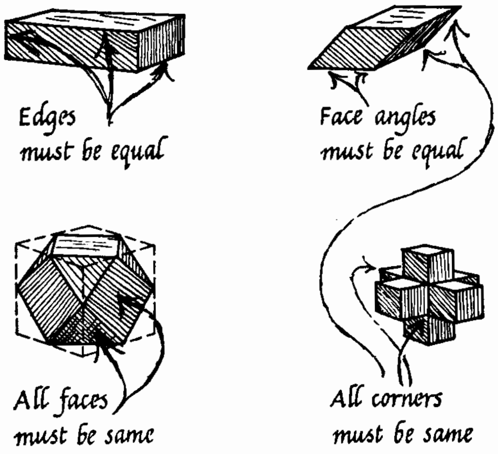
Then try hexagons, and other polygons.

Then return to triangles and carry out similar arguments with triangular faces.

THE RESULT: Only FIVE varieties are possible in our 3-dimensional world. (Fig. 18-3)

(NOTE that these arguments need pencil sketches but can be carried out in your head without cardboard models.)

THE SOLIDS BELOW ARE
NOT REGULAR SOLIDS



THE REGULAR SOLIDS

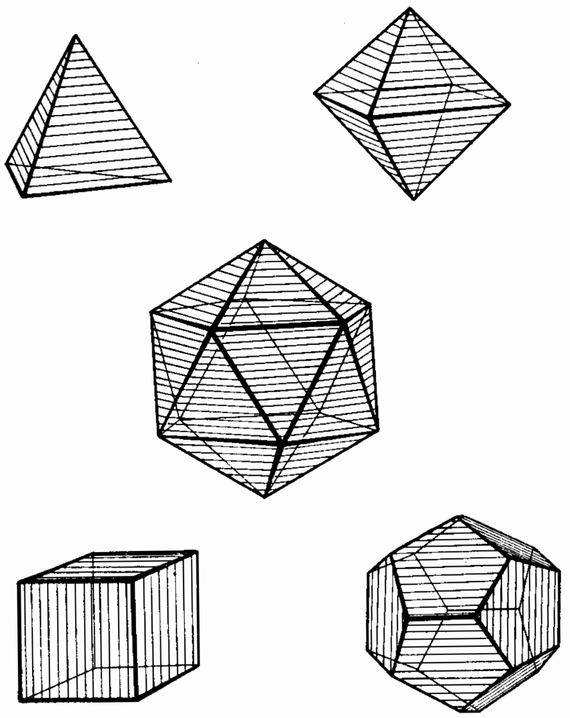


FIG. 18-3.
The five regular solids are drawn after D. Hilbert and S. Cohn-Vossen in *Anschauliche Geometrie* (Berlin: Julius Springer, 1932).

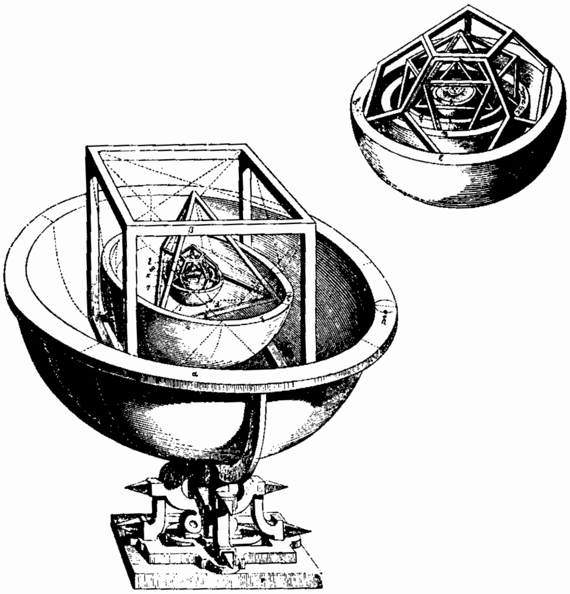


FIG. 18-4. KEPLER'S SCHEME OF REGULAR SOLIDS, FROM HIS BOOK
The relative sizes of planetary orbits were shown by bowls separating one solid from the next. The bowls were not thin shells but were just thick enough to accommodate the *eccentric* orbits of the planets.

The relative radii of the spheres, calculated by geometry, agreed fairly well with the proportions then known for planetary orbits, and Kepler was overjoyed. He said: "The intense pleasure I have received from this discovery can never be told in words. I regretted no more the time wasted; I tired of no labor; I shunned no toil of reckoning, days and nights spent in calculation, until I could see whether my hypothesis would agree with the orbits of Copernicus, or whether my joy was to vanish into air."

We now know the scheme was only a chance success. In later years, Kepler himself had to juggle the proportions by thickening up the bowls to fit the facts; and, when more planets were discovered centuries after, the scheme was completely broken.[4] Yet this "success" sent Kepler on to further, great discoveries.

He published his discovery in a book, including an account of all his unsuccessful trials as well as the successful one. This unusual characteristic appeared in many of his writings. He showed *how* his discoveries were made. He had no fear of damaging his reputation but only wanted to increase human knowledge, so instead of concealing his mistakes he gave a full account of them. "For it is my opinion," he said, "that the occasions by which men have acquired a knowledge of celestial phenomena are not less admirable than the discoveries themselves. . . . If Christopher Columbus, if Magellan, if the Portuguese when they narrate their *wanderings*, are not only excused, but if we do not wish these passages omitted, and should lose much pleasure if they were, let no one blame me for doing the same."

The book also contained an admirable defense of the Copernican system, with good solid reasons in its favor. Young Kepler sent copies of his book to Tycho Brahe and Galileo, who praised it as a courageous beginning. This started Kepler's lifelong friendship with them.[5] In the same book, he made the suggestion that each planet may be pushed along in its orbit by a spoke carrying some influence from the Sun—a vague and improbable idea that later helped him discover his second Law.

Kepler was a Protestant, and he found himself being turned out of his job by Roman Catholic pressure on the administration. Worrying about his future, and anxious to consult Tycho on planetary observations, he travelled across Germany to Prague. Tycho, busy observing Mars, "the difficult planet," wrote to him: "Come not as a stranger but as a friend; come and share in my observations with such instruments as I have with me." While the work of the observatory proceeded, Tycho was turning to detailed "theory," schemes to fit his long series of observations. Kepler was soon set to work on Mars, working with Tycho to find a circular orbit that fitted the facts. Sensitive, and sick, Kepler complained that Tycho treated him as a student and did not share his records freely. Once, driven half crazy by worry, he wrote Tycho a violent letter full of quite unjust reproaches, but Tycho merely argued gently with him. Kepler, repenting, wrote:

"Most Noble Tycho,

How shall I enumerate or rightly estimate your benefits conferred on me? For two months you have liberally and gratuitously maintained me, and my whole family . . . you have done me every possible kindness; you have communicated to me everything you hold most dear. . . . I cannot reflect without consternation that I should have been so given up by God to my own intemperance as to shut my eyes

[4] There *is* a rough empirical rule relating orbit-radii to each other, called Bode's Law; but until recently no reason for it could be found. However, see G. Gamow, *1, 2, 3, . . . Infinity* (New York, Mentor Books, 1953) for a suggested reason.

[5] In a later edition, Kepler took special trouble to avoid any appearance of stealing credit from Galileo. In one of his rejected theories he assumed a planet between Mars and Jupiter. Fearing a careless reader might take this to be a claim anticipating Galileo's discovery of Jupiter's moons, he added a note, saying of his extra planet, "Not circulating round Jupiter like the Medicaean stars. Be not deceived. I never had them in my thoughts."

on all these benefits; that, instead of modest and respectful gratitude, I should indulge for three weeks in continual moroseness towards all your family, in headlong passion and the utmost insolence towards yourself. . . . Whatever I have said or written . . . against your excellency . . . I . . . honestly declare and confess to be groundless, false, and incapable of proof."

When Kepler ended his visit and returned to Germany, Tycho again invited him to join him permanently. Kepler accepted but was delayed by poverty and sickness, and when he reached Prague with no money he was entirely dependent on Tycho. Tycho secured him the position of Imperial Mathematician to assist in the work on the planets.

Tycho died soon after, leaving Kepler to publish the tables. Though he still held the imperial appointment, Kepler had difficulty getting his salary paid and he remained poor, often very poor. At one time he resorted to publishing a prophesying almanac. The idea was abhorrent to him, but he needed the money, and he knew that astrology was the form of astronomy that would pay. For the rest of his life, over a quarter of a century, he worked on the planetary motions, determined to extract the simple secrets he was sure must be there.

The Great Investigation of Mars

When Tycho died, Kepler had already embarked on his planetary investigations, chiefly studying the motion of Mars. What scheme would predict Mars' orbit? Still thinking in terms of circles, Kepler made the planet's orbit a circle round the Sun, with the Sun a short distance off center (like Ptolemy's eccentric Earth). Then he placed an equant point Q off center on the other side, with a spoke from Q to swing the planet around at constant speed. He did not insist, like Ptolemy, on making the eccentric distances CS and CQ equal, but calculated the best proportions for them from some of Tycho's observations. Then he could imagine the planet moving around such an orbit and compare other predicted positions with Tycho's record. He did not know the direction of the line SCQ in space, so he had to make a guess and then try to place a circular orbit on it to fit the facts. Each trial involved long tedious calculations, and Kepler went through 70 such trials before he found a direction and proportions that fitted a dozen observed longitudes of Mars closely. He rejoiced at the results, but then to his dismay the scheme failed badly with Mars' latitudes. He shifted his eccentric distances to a compromise value to fit the latitudes; but, in some parts of the orbit, Mars' position as calculated from his theory disagreed with observation by 8′ (8 sixtieths of one degree). Might not the observations be wrong by this small amount? Would not "experimental error" take the blame? No. Kepler knew Tycho, and he was sure Tycho was never wrong by this amount. Tycho was dead, but Kepler trusted his record. This was a great tribute to his friend and a just one. Faithful to Tycho's memory, and knowing Tycho's methods, Kepler set his belief in Tycho against his own hopeful theory. He bravely set to work to go the whole weary way again, saying that upon these eight minutes he would yet build up a theory of the universe.

It was now clear that a circular orbit would not do. Yet to recognize any other shape of orbit he must obtain an accurate picture of Mars' real orbit from the observations—not so easy, since we only observe the apparent path of Mars from a moving Earth. The true distances were unknown; only angles were measured and those gave a foreshortened compound of Mars' orbital motion and the Earth's. So Kepler attacked the Earth's orbit first, by a method that had all the marks of genius.

Mapping the Earth's Orbit in Space and Time

To map the Earth's orbit around the Sun on a scale diagram, we need many sets of measurements, each set giving the Earth's bearings from *two* fixed points. Kepler took the fixed Sun for one of these, and for the other he took Mars *at a series of times when it was in the same position in its orbit.* He proceeded thus: he marked the "position" of Mars in the star pattern at one opposition (opposite the Sun, overhead at midnight). That gave him the direction of a base line Sun-(Earth)-Mars, SE_1M. Then he turned the pages of Tycho's records to a time *exactly one Martian year later.* (That time of Mars' motion around its orbit was known accurately, from records over centuries.) Then he knew that Mars was in the same position, M, so that SM had the same direction. By now, the Earth had moved on to E_2 in its orbit. Tycho's record of the position of the Mars in the star-pattern gave him the new apparent direction of Mars, E_2M; and the Sun's position gave him the direction E_2S. Then he could calculate the angles of the triangle SE_2M from the record, thus: since he knew the directions E_1M and E_2M (marked on the celestial sphere of stars) he could calculate the angle A between them. Since he knew the directions E_1S and E_2S, he could calculate the angle B, between them. Then on a scale diagram he could choose two points to represent S and M and

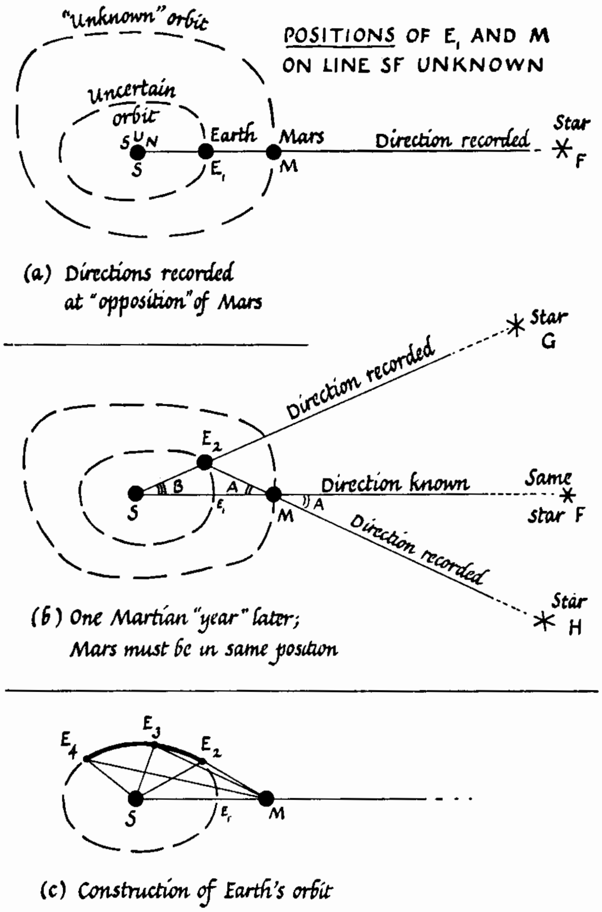


FIG. 18-5. KEPLER'S SCHEME TO PLOT THE EARTH'S ORBIT

locate the Earth's position, E_2, as follows: at the ends of the fixed baseline SM, draw lines making angles A and B and mark their intersection E_2. One Martian year later still, he could find the directions E_3M and E_3S from the records, and mark E_3 on his diagram. Thus Kepler could start with the points S and M and locate E_2, E_3, E_4, . . . enough points to show the orbit's shape.

Then, knowing the Earth's true orbit, he could invert the investigation and plot the shape of Mars' orbit. He found he could treat the Earth's orbit either as an eccentric circle or as slightly oval; but Mars' orbit was far from circular: it was definitely oval or, as he thought, egg-shaped, but he still could not find its mathematical form.

Variable Speed of Planets: Law II

Meanwhile his plot of the Earth's motion in space showed him just how the Earth moves unevenly along its orbit, faster in our winter than in summer. He sought for a law of uneven speed, to replace the use of the equant. His early picture of some pushing influence from the Sun suggested a law to try. He believed that motion needed a force to maintain it, so he pictured a "spoke" from the Sun pushing each planet *along* its orbit, a weaker push at greater distance. He tried (with a confused geometrical scheme) to add up the effects of such pushes from an eccentric Sun; and he discovered a simple law: *the spoke from Sun to planet sweeps out equal areas in equal times.* It does not swing around the Sun with constant speed (as Ptolemy would have liked), but it does have a constancy in its motion: constant rate of sweeping out area (which Ptolemy would probably have accepted). Look at the areas for equal periods, say a month each. When the planet is far from the Sun the spoke sweeps out a long thin triangle in a month; and as the planet approaches the Sun the triangles grow shorter and fatter—the planet moves faster. Later on, when Kepler knew the shape of Mars' orbit he tried the same rule and found it true for Mars too. Here he had a simple law for planetary speeds: each planet moves around the Sun with such speeds that the radius from Sun to planet sweeps out equal areas in equal times. Kepler had only a vague "reason" for it, in terms of solar influences, perhaps magnetic; but he treasured it as a true, simple statement, and used it in later investigations. We treasure it too, and assign a first-class reason to it. We call it Kepler's Second Law. His First Law, discovered soon after, gave the true shape of planetary orbits.

The Orbit of Mars: Law I

When he had plotted Mars' orbit (forty laboriously computed points), Kepler tried to describe its oval shape mathematically. He had endless difficulties—at one time he says he was driven nearly out of his mind by the frustrating complexity. He wrote to the Emperor (to encourage finances), in

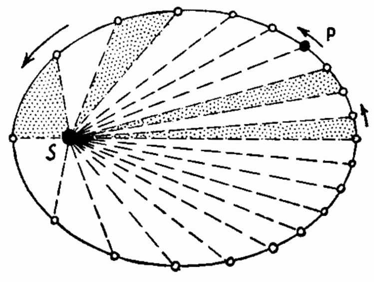


FIG. 18-6. KEPLER'S DISCOVERIES FOR MARS
An ellipse with the Sun in one focus fits the orbit of Mars. The spoke from Sun to Planet sweeps out equal areas in equal times. The positions marked here show planet's positions at equal intervals of time, 1/20 of its "year" apart. The planet moves with such speeds that all the sectors marked here—a few of them shaded—have equal areas.

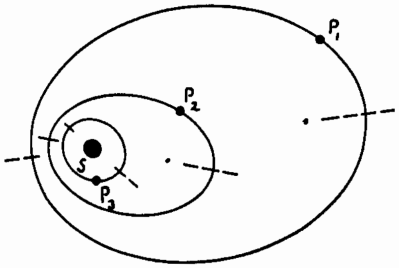


Fig. 18-7. A Solar System with Elliptical Orbits Around a Common Sun
(The planets' orbits in our own Solar System have much smaller eccentricities. But some comets move in elliptical orbits with great eccentricity.)

his grandiose style: "While triumphing over Mars, and preparing for him, as for one already vanquished, tabular prisons and equated excentric fetters, it is buzzed here and there that my victory is vain, and that the war is raging anew. For the enemy left at home a despised captive has burst all the chains of the equations, and broken forth from the prisons of the tables."

Finally, he found the true orbit sandwiched between an eccentric circle that was too wide and an inscribed ellipse that was too narrow. Both disagreed with observation, the circle by $+8'$ at some places, the inner ellipse by $-8'$. He suddenly saw how to compromise half way between the two, and found that gave him an orbit that is *an ellipse with the Sun in one focus.* He was so delighted with his final proof that this would work that he decorated his diagram with a sketch of victorious Astronomy (Fig. 18-8). At last he knew the true orbit of Mars.[6] A similar rule holds for the Earth and other planets. This is his First Law.

[6] It may seem strange that he did not think of an ellipse earlier. It was a well-known oval, studied by the Greeks as one of the sections of a cone. But then *we* know the answer. Besides, ellipses were not so important then. It was Kepler who added greatly to their fame. (An ellipse is easy to draw with a loop of string and two thumb-tacks. If you have never tried making one for yourself you should do so. This is an amusing experiment which will show you a property of ellipses that is valuable in optics.)

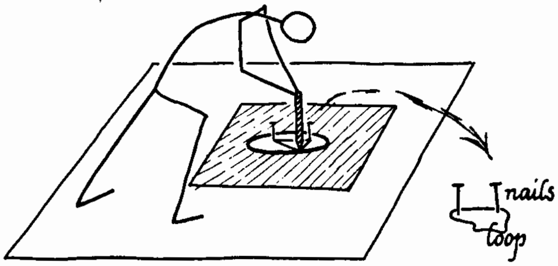


Fig. 18-9.
Drawing an Ellipse, with a loop of thread and two nails

Law III

Kepler had then extracted two great "laws" from Tycho's tables, by his fearless thinking and untiring work. He continued to brood on one of his early questions: what connection is there between the sizes of the planets' orbits and the times of their "years"? He now knew the average radii[7] of the orbits; the times of revolution ("years") had long been known. (As the Greeks surmised, the planets with the longest "years" have the largest orbits.) He felt sure there was some relation between radius and time. He must have made and tried many a

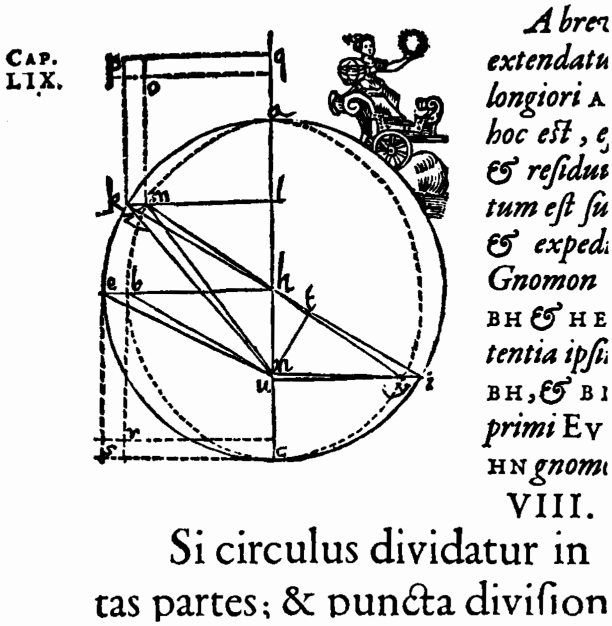


Fig. 18-8.
Kepler's Triumphant Diagram, From his Book on Mars
When he succeeded in proving that an ellipse with the Sun in one focus could replace an oscillating circular orbit and maintain an "equal area" law, Kepler added a sketch of Victorious Astronomy, to show his delight and to emphasize the importance of the proof.

guess, some of them sterile ones like his early scheme of the five regular solids or wild mystical ones like his speculation of musical chords for the planets. Fortunately there *is* a connection between radii and times, and Kepler lived to experience the joy of finding it. He found that the fraction R^3/T^2 is the same for all the planets, where R is the planet's average orbit-radius, and T is the planet's "year," measured in our days. See the table.

[7] Assuming circular orbits, Copernicus made rough estimates, and Tycho made better ones. Kepler knew these when he tried his strange scheme of regular solids, and he traded on their roughness to let his test of that theory seem "successful."

PLANETARY DATA — TEST OF KEPLER'S THIRD LAW
(These are modern data, more accurate than Kepler's)

Planet	*Radius of planet's orbit* R (*miles*)	*Time of revolution* (*planet's "year"*) T (*days*)	R^3 (*miles*)^3	T^2 (*days*)^2	$\frac{R^3}{T^2}$ $\frac{(miles)^3}{(days)^2}$
Mercury	3.596×10^7	87.97	46.50×10^{21}	7739.	6.009×10^{18}
Venus	6.720×10^7	224.7	303.5×10^{21}	50490.	6.011×10^{18}
Earth	9.290×10^7	365.3	801.8×10^{21}	133400.	6.010×10^{18}
Mars	14.16×10^7	687.0	$2839. \times 10^{21}$	472100.	6.015×10^{18}
Jupiter	48.33×10^7	4332.	$112900. \times 10^{21}$	18770000.	6.015×10^{18}
Saturn	88.61×10^7	10760.	$695700. \times 10^{21}$	115800000.	6.008×10^{18}

The test of Kepler's guess is shown in the last column

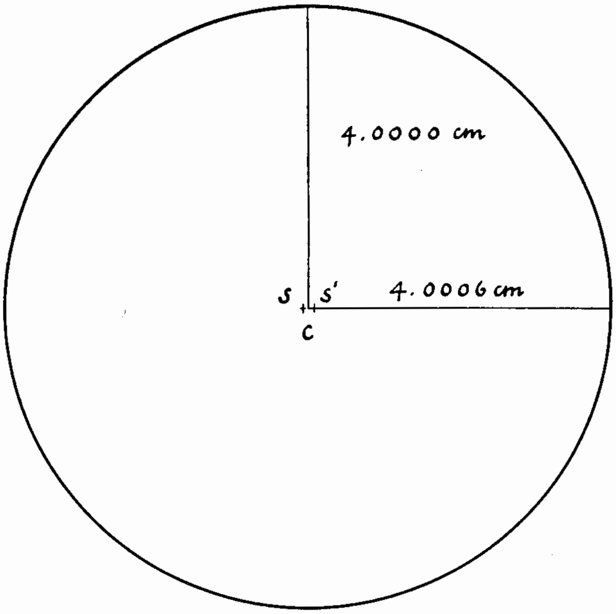


FIG. 18-10. ELLIPSE: THE EARTH'S ORBIT DRAWN TO SCALE
The actual eccentricity of planetary orbits is very small. The orbits are almost circles, yet Tycho's observations enabled Kepler to show that they are not circles but ellipses. The sketch above shows the Earth's orbit drawn to scale. If a 4.0000 centimeter line is used, as here, to represent the minimum radius, which is really some 93,000,000 miles, the maximum radius needs a line 4.0006 centimeters long. The eccentricity of Mars' orbit is over thirty times as big, but even then the ratio of radii is only 1.0043 to 1.0000. Mercury is the only planet with a much greater eccentricity of orbit, with radii in proportion 1.022 to 1.000. Even this eccentricity of orbit seems small, but it is sufficient to involve Mercury in such speed changes around the orbit that Relativity mechanics predicts a very slow slewing around of the orbit—a precession of only 1/80 of a degree per *century*, discovered and measured long before the Relativity prediction!

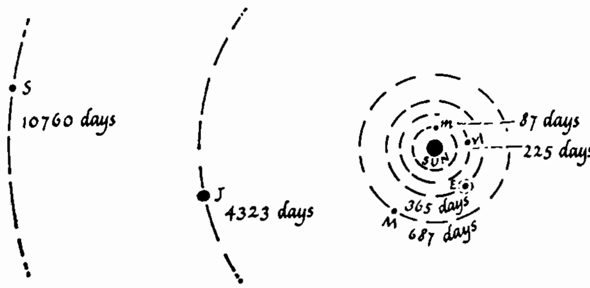


FIG. 18-11a. ? RELATIONSHIP BETWEEN *RADIUS* AND *"YEAR"* FOR PLANETARY ORBITS ?
(Planetary orbits roughly to scale.)

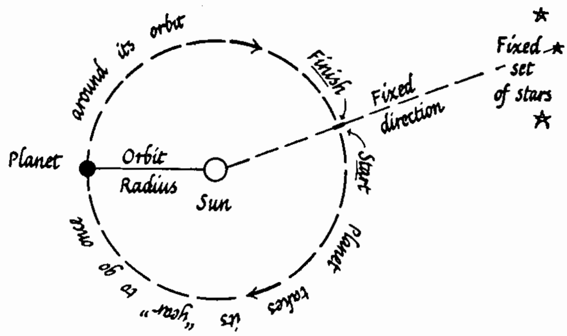


FIG. 18-11b. PLANET'S "YEAR"
The planet's year is the time it takes to go once around its orbit. This is the time-interval from the moment when its direction hits some standard mark in the star-pattern until it returns to the same mark. (The Earth moves too. An allowance for the Earth's motion must be made when extracting the planet's true year from observations.)

Again he was overjoyed at wresting a divine secret from Nature by brilliant guessing and patient trial. He said:

"What I prophesied two-and-twenty years ago, as soon as I discovered the five solids among the heavenly orbits—what I firmly believed long before I had seen Ptolemy's "Harmonies"—what I had promised my friends in the title of this book, which I named before I was sure of my discovery—what sixteen years ago, I urged as a thing to be sought—that for which I joined Tycho Brahe, for which I settled in Prague, for which I have devoted the best part of my life to astronomical contemplations, at length I have brought to light, and recognized its truth beyond my most sanguine expectations. It is not eighteen months since I got the first glimpse of light, three months since the dawn, very few days since the unveiled sun, most admirable to gaze upon, burst upon me. Nothing holds me . . . the die is cast, the book is written, to be read either now or by posterity, I care not which; it may well wait a century for a reader, as God has waited six thousand years for an observer."

Kepler's Laws

These investigations took years of calculating, changing, speculating, calculating. . . . Kepler discovered—among other "harmonies" that he valued—three great laws that are clear and true. Here they are:

LAW I — EACH PLANET MOVES IN AN ELLIPSE WITH THE SUN IN ONE FOCUS.

LAW II — THE RADIUS VECTOR (LINE JOINING SUN TO PLANET) SWEEPS OUT EQUAL AREAS IN EQUAL TIMES.

LAW III — THE SQUARES OF THE TIMES OF REVOLUTION (OR YEARS) OF THE PLANETS ARE PROPORTIONAL TO THE CUBES OF THEIR AVERAGE DISTANCES FROM THE SUN.
(Or R^3/T^2 is the same for all the planets)

Once guessed, the first two laws could be tested with precision with available data; so Kepler could make sure he had guessed right. Law III was tested in its discovery. Only relative values of orbit-radii were needed.

Kepler had done a great piece of work. He had discovered the laws that Newton linked with universal gravitation. Of course that was not what Kepler thought he was doing. "He was not tediously searching for empirical rules to be rationalised by a coming Newton. He was searching for ultimate causes, the mathematical harmonies in the mind of the Creator."[8] He emerged with no general reason for his ellipses and mathematical relationships; but he delighted in their truth.

Guessing the Right Law

Guessing the third law was a matter of finding a numerical relationship which would hold for several pairs of numbers. An infinite variety of "wrong" guesses can be made to fit a limited supply of data, in this case values of T and R for only six planets. Many such guesses that succeed with six planets fail when applied to a seventh planet (Uranus, discovered later). Of those that still succeed, many would fail if tried on an eighth planet (Neptune). So trials with more and more sets of data can help to remove "wrong" guesses, leaving the "right" one. But in what sense is the "right" one right? Some of us believe there is a really true story behind the things we see in Nature. Kepler, Galileo, and Newton probably thought like that. Others now say that the right rule is merely (*a*) *the rule that applies most generally* (for example, to the greatest variety of planets). In this sense Kepler's R^3/T^2 guess was right because it applies to later-discovered planets and to other systems such as Jupiter's moons. His five-regular-solids rule was wrong, because it did not agree well with data for the original six planets and failed completely when required to deal with more than six. And, they say, the right rule is (*b*) *the rule that fits best into a theoretical framework which ties together a variety of knowledge of Nature.* If that theory has been manufactured just to deal with the problem in hand, then (*b*) is nonsense—it would merely say that the rule is right because it agrees with its own theory constructed to agree with it. We call that an *ad hoc* theory. If, however, the theory connects the problem in hand to other natural knowledge, then (*b*) is a cogent recommendation. Newton, guessing at universal gravitation, made a theory that connects falling bodies and the Moon's motion and planetary motion and tides, etc. He showed that Kepler's Law III (as well as the other two) was a necessary deduction from this theory. Thus Kepler's R^3/T^2 rule seems "right" on both scores, (*a*) and (*b*), *generalness* and *agreement with wide theory.* It might have been a "wrong" guess, waiting like the early "five-regular-solids" law for more data to refute it and for theory to fail to "predict"[9] it.

[8] Sir William Dampier, *op.cit.*

[9] Scientists use "predict" in this way, but it is an unfortunate choice of word. Here it means "coordinate with other knowledge."

A Fictitious "Kepler Problem"

To see something of the hazards involved in an investigation like Kepler's let us trace through a specimen problem using imaginary data, with a fictitious relationship. Suppose you have invented a planetary puzzle and know the scheme you have used, but ask me to try to find the scheme. You present me with the following data.

Data			Problem
"Planet"	R	T	What is the "law" connecting R and T?
A	1	3	
B	2	6	
C	4	18	

You know the scheme, since you have invented it. (It is not an inverse square law system: the "planets" are not real ones!) In fact, you got T by squaring R and adding 2. That is, you *chose* the relation $T = R^2 + 2$ and used it. (Make sure our data fit this formula.) So if a new planet D is discovered with $R = 5$ it will have $T = 5^2 + 2$, or 27. Suppose you give me the data for A, B, C (holding D up your sleeve). In looking for a rule, I try to find some algebraic combination of T and R which will be the *same for each of these planets.* Starting with planets A and B, I notice that T/R is 3/1 for A, 6/2 for B, the same for both. Hoping I have found the right rule (T/R the same for all), I try this on planet C. For C, T/R is 18/4 and this is not the same as 3/1. I must therefore reject this simple guess. In trying other schemes which give the same answer for planets A and B, I find several more which fail for C. But presently I find that I get the same answer for planets A and B if I proceed thus: I divide R into 8 and add 7 times R and subtract T; that is, I find the value of $8/R + 7R - T$.

For planet A, $8/1 + 7 \times 1 - 3 = 12$;
and for planet B, $8/2 + 7 \times 2 - 6 = 12$.

So the answer is the same, 12, for both A and B. Trying the same rule on planet C,
I have $8/4 + 7 \times 4 - 18 = 12$ again.

So I am delighted to find the rule works for C and A and B. Confident that I have got the right rule, I plan to publish it, but you then divulge the data for planet D: $R = 5$ and $T = 27$. Trying my rule on planet D,
I obtain $8/5 + 7 \times 5 - 27 = 9.6$.

After asking you whether your data might be wrong enough to excuse the difference between 9.6 and 12.0, I start all over again. If I am lucky as well as patient, I may hit upon a scheme such as this: add 2 to the square of R and divide by T. This yields an answer 1.000 for all four planets, A, B, C, D.[10] Therefore it has a better chance of being the right rule than the others. Tests on more data would improve its reputation further and if some general theory could endorse it I might feel sure I had the right rule. Summing up this investigation in a table, we have

"PLANET"	DATA		ATTEMPTS TO OBTAIN CONSTANT NUMBERS		
			1st Trial	Nth Trial	Qth Trial
	R	T	$\frac{T}{R}$	$\frac{8}{R} + 7R - T$	$\frac{R^2 + 2}{T}$
A	1	3	3	12	1
B	2	6	3	12	1
C	4	18	4.5	12	1
D	5	27	5.4	9.6	1
e	3	11	3.667	12.67	1

Note that at the last moment another "planet" has been discovered, e, which is so small that it was not noticed before. It too fits with the final rule (of course it does, in this game, since *you* manufactured its data by using your private knowledge of that rule), and it fails to fit with the earlier rules. Notice, however, that it nearly fits with the second rule, giving 12.67 instead of 12.00. If the data for planet e had been available when I was working on my second rule, should I not have been tempted to say "12.67 is near enough; the difference is due to experimental error"?

Kepler's Writing

Kepler wrote many books and letters setting forth his discoveries in detail, describing failures as well as successes. His account of his Laws is immersed in much mystical writing about other discoveries and ideas: planetary harmonies, schemes of magnetic influence, hints about gravitation, and a continuing delight in his earliest scheme of the five regular solids. Remember Kepler did not know the "right answers." He had no idea which of his theories would be validated by later discoveries and thought. He finally managed to get the Rudolphine tables printed—paying some of the cost himself, which he could hardly afford—so that at last really good astronomical data were available. Among his own books, he wrote a careful fairly popular book on general astronomy in which he explained the Copernican theory and described his own discoveries. The book was at once suppressed by the Church authorities, leaving him all the poorer by making it hard to get any of his books published and sold.

Comments on Kepler

"When Kepler directed his mind to the discovery of a general principle, he . . . never once lost sight of the explicit object of his search. His imagination, now unreined, indulged itself in the creation and invention of various hypotheses. The most plausible,

[10] There is no special virtue in the answer being 1.000. If I divide by $5T$ instead of by T the answers would all be 0.200, but the essential story is unchanged.

These data provide a problem somewhat like the one that faced Kepler when he had planetary orbit data but had not guessed his third law. There is a fairly simple relationship between N and v.

Can you find this relationship? Try this, as Kepler would, with courage and care, without any help from a theory or a book. If you find the relationship, show how closely the data fit it. Of course, the original experimenters had an advantage over you; they knew what relation to try first—but then they had to do a difficult experiment. In these difficult experiments of counting *single atoms* as they bounce away from the gold, you must not expect great accuracy; so, unlike Kepler's, your constant may wobble by 10% but not in any particular direction.