Guadarrama Rodriguez Vanesa.

Elipse

Sea la elipse: $-7.25x^2+12xy-24y^2-19x+132y=165.75$

$p^tAp+2b^tp+\gamma=0$

Sea $A=\begin{pmatrix}-7.25 & 6\\ 6 & -24\end{pmatrix}$

$b=\begin{pmatrix}-\frac{19}{2}\\ 66\end{pmatrix}$ y $\gamma=165.75$

Calculando A^{-1}

$$A^{-1}=\frac{1}{174-36}\begin{pmatrix}-24 & -6\\ -6 & -7.25\end{pmatrix}$$

$$=-\frac{1}{138}\begin{pmatrix}24 & 6\\ 6 & 7.25\end{pmatrix}$$

$$=\begin{pmatrix}-\frac{24}{138} & -\frac{6}{138}\\ -\frac{6}{138} & -\frac{7.25}{138}\end{pmatrix}$$

Buscando P_0 = centro de la elipse

$$P_0=-A^{-1}b$$

$$P_0=\begin{pmatrix}\frac{24}{138} & \frac{6}{138}\\ \frac{6}{138} & \frac{7.25}{138}\end{pmatrix}\begin{pmatrix}-\frac{19}{2}\\ 66\end{pmatrix}$$

$$\boxed{P_0=\begin{pmatrix}\frac{28}{23}\\ \frac{281}{92}\end{pmatrix}}$$ Centro de la cónica (elipse)

Evaluar P_0 en la cónica para obtener γ

$$-7.25\left(\frac{28}{23}\right)^2+12\left(\frac{28}{23}\right)\left(\frac{281}{92}\right)-24\left(\frac{281}{92}\right)^2$$
$$-19\left(\frac{28}{23}\right)+132\left(\frac{281}{92}\right)-165.75=\boxed{\frac{2233}{92}}$$

$$P(\lambda)=\det(A-\lambda I)=0$$

$$=\det\begin{pmatrix}-7.25-\lambda & 6\\ 6 & -24-\lambda\end{pmatrix}=0$$

$$(\lambda+7.25)(\lambda+24)-36=0$$
$$\lambda^2+\frac{125}{4}\lambda+174-36=0$$
$$\lambda^2+\frac{125}{4}\lambda+138=0$$

Resolviendo para λ

$$\lambda_{1,2}=\frac{-\frac{125}{4}\pm\sqrt{\left(\frac{125}{4}\right)^2-4(1)(138)}}{2}$$

$$\lambda_{1,2}=\frac{-\frac{125}{4}\pm\sqrt{\frac{15625}{16}-552}}{2}$$

$$\lambda_{1,2}=\frac{-\frac{125}{4}\pm\sqrt{\frac{6793}{16}}}{2}$$

$$\lambda_{1,2}=-\frac{125}{8}\pm\sqrt{\frac{6793}{64}}$$

$$\lambda_1=-\frac{125}{8}+\frac{\sqrt{6793}}{8}=\boxed{-5.3225}$$

$$\lambda_2=-\frac{125}{8}-\frac{\sqrt{6793}}{8}=\boxed{-25.9275}$$

Obteniendo los vectores u_1 y u_2 a partir de λ_1 y λ_2

Para $\lambda_1=-5.3225$

$$\begin{cases}(-7.25-\lambda)x+6y=0 & (1)\\ 6x+(-24-\lambda)y=0 & (2)\end{cases}$$

Utilizando (1) y λ_1

$$(-7.25+5.3225)x+6y=0$$
$$-\frac{771}{400}x+6y=0$$

Sea $u_1=\begin{pmatrix}1\\ \frac{257}{800}\end{pmatrix}$

y $u_2=\begin{pmatrix}\frac{257}{800}\\ -1\end{pmatrix}$

normalizando a μ_1 y μ_2

$$\|\mu_1\| = \sqrt{1^2 + \left(\frac{257}{800}\right)^2} = \sqrt{1.1032}$$

$$\mu_1 = \begin{pmatrix} \frac{1}{\sqrt{1.1032}} \\ \frac{\frac{257}{800}}{\sqrt{1.1032}} \end{pmatrix}$$

y $$\mu_2 = \begin{pmatrix} \frac{\frac{257}{800}}{\sqrt{1.1032}} \\ -\frac{1}{\sqrt{1.1032}} \end{pmatrix}$$

$$\mu_1^2 - \mu_2^2 = \begin{pmatrix} \frac{1}{1.1032} \\ \frac{0.1032}{1.1032} \end{pmatrix} - \begin{pmatrix} \frac{0.1032}{1.1032} \\ +\frac{1}{1.1032} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1121}{1379} \\ -\frac{1121}{1379} \end{pmatrix}$$

$$\mu_2^2 + \mu_1^2 = \begin{pmatrix} \frac{0.1032}{1.1032} \\ +\frac{1}{1.1032} \end{pmatrix} - \begin{pmatrix} \frac{1}{1.1032} \\ \frac{0.1032}{1.1032} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1121}{1379} \\ \frac{1121}{1379} \end{pmatrix}$$

$$\delta = \begin{pmatrix} \frac{1121}{1379} & -\frac{1121}{1379} \\ -\frac{1121}{1379} & \frac{1121}{1379} \end{pmatrix}$$

Eje de simetría

* $P_0 + \alpha \vec{\mu_1} = 0$

$$\begin{pmatrix} \frac{28}{23} \\ \frac{281}{92} \end{pmatrix} + \alpha \begin{pmatrix} \frac{1}{\sqrt{1.1032}} \\ \frac{257}{800\sqrt{1.1032}} \end{pmatrix} = 0$$

$$x = \frac{28}{23} + \alpha\left(\frac{1}{\sqrt{1.1032}}\right)$$

$$y = \frac{281}{92} + \alpha\left(\frac{257}{800\sqrt{1.1032}}\right)$$

$$\frac{x - \frac{28}{23}}{\frac{1}{\sqrt{1.1032}}} = \alpha = \frac{y - \frac{281}{92}}{\frac{257}{800\sqrt{1.1032}}}$$

$$\frac{x - \frac{28}{23}}{0.9521} = \frac{y - \frac{281}{92}}{0.3059}$$

$$0.3059x - \frac{931}{2500} = 0.9521y - 2.9080$$

$$\boxed{0.3059x - 0.9521y + 2.5356 = 0}$$ eje mayor

* $P_0 + \beta \vec{\mu_2} = 0$

$$\begin{pmatrix} \frac{28}{23} \\ \frac{281}{92} \end{pmatrix} + \beta \begin{pmatrix} \frac{257}{800\sqrt{1.1032}} \\ -\frac{1}{\sqrt{1.1032}} \end{pmatrix} = 0$$

$$x = \frac{28}{23} + \beta\left(\frac{257}{800\sqrt{1.1032}}\right)$$

$$y = \frac{281}{92} + \beta\left(-\frac{1}{\sqrt{1.1032}}\right)$$

$$\frac{x - \frac{28}{23}}{0.3059} = \beta = \frac{y - \frac{281}{92}}{-0.9521}$$

$$\frac{x - \frac{28}{23}}{0.3059} = \frac{y - \frac{281}{92}}{-0.9521}$$

$$-0.9521x + 1.1591 = 0.3059y - 0.9343$$

Eje menor $$\boxed{0 = 0.9521x + 0.3059y - 2.0934}$$

Vector (En gráfica) Eje mayor

$$\vec{\mu} = P_0 - \mu_1$$

$$= \begin{pmatrix} \frac{28}{23} \\ \frac{281}{92} \end{pmatrix} - \begin{pmatrix} \frac{1}{\sqrt{1.1032}} \\ \frac{257}{800\sqrt{1.1032}} \end{pmatrix} = \begin{pmatrix} 0.2653 \\ 2.7484 \end{pmatrix}$$

$$\vec{v} = P_0 - \mu_2$$

$$= \begin{pmatrix} \frac{28}{23} \\ \frac{281}{92} \end{pmatrix} - \begin{pmatrix} \frac{257}{800\sqrt{1.1032}} \\ -\frac{1}{\sqrt{1.1032}} \end{pmatrix} = \begin{pmatrix} 0.9115 \\ 4.0064 \end{pmatrix}$$

Elipse.ggb

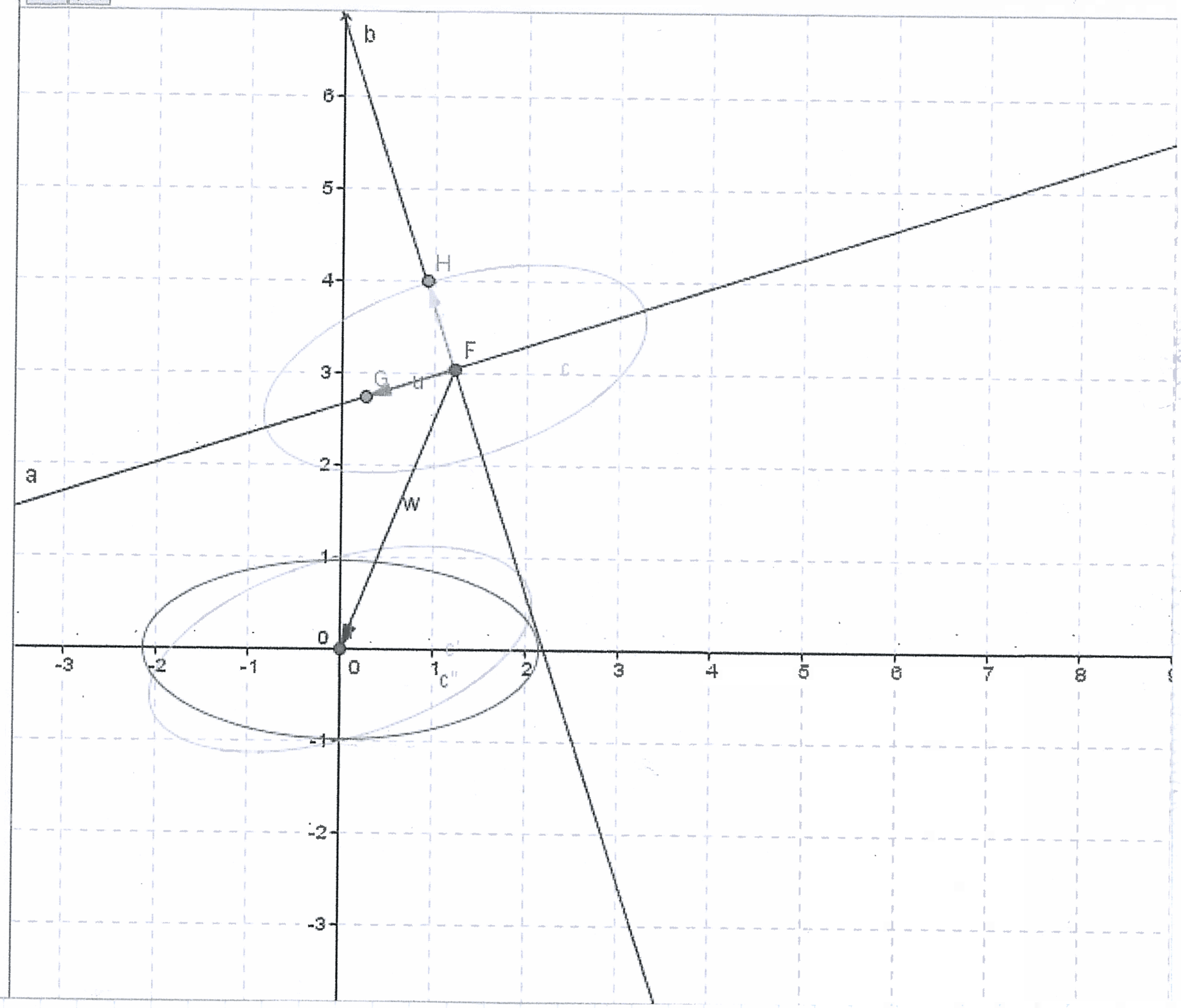
Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Abrir ses

Vista Algebraica

- Cónica
  - c: $-7.25x^2 + 12x\,y - 24y^2 - 19x + 132y = 165.75$
  - c': $-7.25x^2 + 12x\,y - 24y^2 = -24.27$
  - c'': $-5.32x^2 - 25.93y^2 = -24.27$
- Número
  - e = 2.14
  - f = 0.97
- Punto
  - A = (0, 3.56)
  - B = (1, 2)
  - C = (3, 3)
  - D = (3, 4)
  - E = (1, 4)
  - F = (1.22, 3.05)
  - G = (0.26, 2.75)
  - H = (0.91, 4)
  - I = (0, 0)
  - J = (1.03, 0.33)
  - K = (1.43, 0)
- Recta
  - a: $0.31x - 0.95y = -2.54$
  - b: $0.95x + 0.31y = 2.09$
  - d: $0.31x - 0.95y = 0$
- Vector
  - $u = \begin{pmatrix} -0.96 \\ -0.31 \end{pmatrix}$
  - $v = \begin{pmatrix} -0.3 \\ 0.94 \end{pmatrix}$
  - $w = \begin{pmatrix} -1.22 \\ -3.05 \end{pmatrix}$
- Ángulo
  - $\alpha = 17.81°$

Vista Gráfica



Quevavama Rodríguez Vanesa

- Ec. de la cónica transformada a su centro, que tiene forma: $\tilde{p}^t A \tilde{p} + \tilde{\gamma} = 0$

Sabemos $\tilde{\gamma} = \frac{2233}{92}$

ent. $\tilde{\mathcal{C}} = \tilde{p}^t A \tilde{p} + \tilde{\gamma} = 0$

$= (\tilde{x}, \tilde{y}) \begin{pmatrix} -7.25 & 6 \\ 6 & -24 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} + \frac{2233}{92} = 0$

$$\boxed{\tilde{\mathcal{C}} : -7.25\tilde{x}^2 + 12\tilde{x}\tilde{y} - 24\tilde{y}^2 + \frac{2233}{92} = 0}$$

- Matrices de simetría

$$S = \begin{pmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ 2\alpha\beta & \beta^2 - \alpha^2 \end{pmatrix}$$

$$\mu_1 = \begin{pmatrix} \frac{1}{\sqrt{1.1032}} \\ \frac{257}{800\sqrt{1.1032}} \end{pmatrix} \quad y \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\mu_2 = \begin{pmatrix} \frac{257}{800\sqrt{1.1032}} \\ -\frac{1}{\sqrt{1.1032}} \end{pmatrix} \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$S_1 = \begin{pmatrix} \frac{1}{1.1032} - \frac{66049}{706048} & \frac{514}{882.56} \\ \frac{514}{882.56} & \frac{66049}{706048} - \frac{1}{1.1032} \end{pmatrix}$$

$$S_2 = \begin{pmatrix} \frac{66049}{706048} - \frac{1}{1.1032} & -\frac{514}{882.56} \\ -\frac{514}{882.56} & \frac{1}{1.1032} - \frac{66049}{882.56} \end{pmatrix}$$

- Matriz Q tal que $\tilde{p} = Q\hat{p}$ transforme $\tilde{p}^t A \tilde{p} + \tilde{\gamma} = 0$ a $\hat{p}^t D \hat{p} + \tilde{\gamma} = 0$

$\tilde{p} = Q\hat{p}$

$(Q\hat{p})^t A (Q\hat{p}) + \tilde{\gamma} = 0$

$\hat{p}^t Q^t A Q \hat{p} + \tilde{\gamma} = 0$

$\hat{p}^t D \hat{p} + \tilde{\gamma} = 0$ con $D = Q^t A Q$

$QD = AQ$

$QDe_1 = AQe_1$

$Qe_1 = Aq_1$

ent. $D = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$Q = (\mu_1 | \mu_2)$

$\|\mu_1\| = \|\mu_2\| = 1$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{1.1032}} & \frac{257}{800\sqrt{1.1032}} \\ \frac{257}{800\sqrt{1.1032}} & -\frac{1}{\sqrt{1.1032}} \end{pmatrix}$$

$$D = \begin{pmatrix} -5.3225 & 0 \\ 0 & -25.9275 \end{pmatrix}$$

Calculando

$\hat{p}^t D \hat{p} + \hat{\gamma} = 0$

$0 = (\hat{x}, \hat{y}) \begin{pmatrix} -5.3225 & 0 \\ 0 & -25.9275 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + \frac{2233}{92}$

$$\boxed{0 = -5.3225\hat{x}^2 - 25.9275\hat{y}^2 + \frac{2233}{92}}$$

Ecuación de la elipse en el origen y con ejes de simetría = eje "x" y eje "y"

- Tamaño de los ejes

$$5.3225\hat{x}^2 + 25.9275\hat{y}^2 = \frac{2233}{92}$$

$$\frac{2233}{92}$$

$$\frac{5.3225\hat{x}^2}{24.2717} + \frac{25.9275\hat{y}^2}{24.2717} = 1$$

$$\frac{\hat{x}^2}{\frac{24.2717}{5.3225}} + \frac{\hat{y}^2}{\frac{24.2717}{25.9275}} = 1$$

$\therefore \; a = \sqrt{\frac{24.2717}{5.3225}} = 2.135\,u$

$b = \sqrt{\frac{24.2717}{25.9275}} = 0.9675\,u$

Semieje a $= 2.135\,u$

Semieje b $= 0.9675\,u$

Guadarrama Rodríguez Vanesa

Hipérbola

Sea la hipérbola: $1.6x^2 + 18.53xy - 6.16y^2 - 50.98x + 24.57y = 24.51$

$$p^tAp + 2b^tp + \gamma = 0$$

Sea $A = \begin{pmatrix} 1.6 & \frac{1853}{200} \\ \frac{1853}{200} & -6.16 \end{pmatrix}$

$b = \begin{pmatrix} -\frac{2549}{100} \\ \frac{2457}{200} \end{pmatrix}$ y $\gamma = -24.51$

Calculando A^{-1}

$$A^{-1} = \frac{1}{-95.7}\begin{pmatrix} -6.16 & -\frac{1853}{200} \\ -\frac{1853}{200} & 1.6 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6.16}{95.7} & \frac{1853}{(95.7)(200)} \\ \frac{1853}{(95.7)(200)} & -\frac{1.6}{95.7} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{28}{435} & \frac{1853}{19140} \\ \frac{1853}{19140} & -\frac{16}{957} \end{pmatrix}$$

Buscando P_0 de la hipérbola

$$P_0 = -A^{-1}b$$

$$P_0 = \begin{pmatrix} -\frac{28}{435} & -\frac{1853}{19140} \\ -\frac{1853}{19140} & \frac{16}{957} \end{pmatrix}\begin{pmatrix} -\frac{2549}{100} \\ \frac{2457}{200} \end{pmatrix}$$

$$\boxed{P_0 = \begin{pmatrix} 0.4514 \\ 2.6732 \end{pmatrix}}$$ centro de la cónica.

Evaluar P_0 en la cónica para obtener $\tilde{\gamma}$

$$1.6(0.4514)^2 + 18.53(0.4514)(2.6732) - 6.16(2.6732)^2 - 50.98(0.4514) + 24.57(2.6732) - 24.51 = \boxed{-3.1753}$$

$$P(\lambda) = \det(A - \lambda I) = 0$$

$$= \det\begin{pmatrix} 1.6-\lambda & \frac{1853}{200} \\ \frac{1853}{200} & -6.16-\lambda \end{pmatrix} = 0$$

$$(\lambda - 1.6)(\lambda + 6.16) - 85.8402 = 0$$

$$\lambda^2 + 4.56\lambda - \frac{1232}{125} - 85.8402 = 0$$

$$\lambda^2 + 4.56\lambda - 95.6962 = 0$$

Resolviendo para λ

$$\lambda_{1,2} = \frac{-4.56 \pm \sqrt{\frac{12996}{625} + 382.7848}}{2}$$

$$\lambda_{1,2} = \frac{-4.56 \pm \sqrt{403.5784}}{2}$$

$$\lambda_1 = \frac{-4.56 + \sqrt{403.5784}}{2} = \boxed{7.7646}$$

$$\lambda_2 = \frac{-4.56 - \sqrt{403.5784}}{2} = \boxed{-12.3246}$$

Obteniendo los vectores $\vec{u}_1$ y u_2 a partir de λ_1 y λ_2

Para $\lambda_1 = 7.7646$

$$\begin{cases} (1.6-\lambda)x + \frac{1853}{200}y = 0 & (1) \\ \frac{1853}{200}x + (-6.16-\lambda)y = 0 & (2) \end{cases}$$

Utilizando (1) y λ_1

$$(1.6 - 7.7646)x + \frac{1853}{200}y = 0$$

$$-\frac{30823}{5000}x + \frac{1853}{200}y = 0$$

$$\vec{u}_1 = \begin{pmatrix} 1.503 \\ 1 \end{pmatrix}$$

y $\vec{u}_2 = \begin{pmatrix} -1 \\ 1.503 \end{pmatrix}$

Normalizando a u_1 y u_2

$$\|u\| = \sqrt{1 + 2.259}$$

$$= \sqrt{3.259}$$

$$\vec{\mu}_1 = \begin{pmatrix} \frac{1.503}{\sqrt{3.259}} \\ \frac{1}{\sqrt{3.259}} \end{pmatrix}$$

y

$$\vec{\mu}_2 = \begin{pmatrix} -\frac{1}{\sqrt{3.259}} \\ \frac{1.503}{\sqrt{3.259}} \end{pmatrix}$$

Eje de simetría

$* \quad P_0 + \alpha \mu_1 = 0$

$$\begin{pmatrix} 0.4514 \\ 2.6732 \end{pmatrix} + \alpha \begin{pmatrix} \frac{1.503}{\sqrt{3.259}} \\ \frac{1}{\sqrt{3.259}} \end{pmatrix} = 0$$

$$x = 0.4514 + \alpha\left(\frac{1.503}{\sqrt{3.259}}\right)$$
$$y = 2.6732 + \alpha\left(\frac{1}{\sqrt{3.259}}\right)$$

$$\frac{x-0.4514}{0.8326} = \alpha = \frac{y-2.6732}{0.5539}$$

$$\frac{x-0.4514}{0.8326} = \frac{y-2.6732}{0.5539}$$

$$0.5539x - 0.25 = 0.8326y - 2.2257$$
$$0.5539x - 0.8326y - 0.25 + 2.2257 = 0$$
$$\boxed{0.5539x - 0.8326y + 1.9757 = 0} \text{ Eje}$$

$* \quad P_0 + \beta \mu_2 = 0$

$$\begin{pmatrix} 0.4514 \\ 2.6732 \end{pmatrix} + \beta \begin{pmatrix} -\frac{1}{\sqrt{3.259}} \\ \frac{1.503}{\sqrt{3.259}} \end{pmatrix} = 0$$

$$x = 0.4514 + \beta\left(-\frac{1}{\sqrt{3.259}}\right)$$
$$y = 2.6732 + \beta\left(\frac{1.503}{\sqrt{3.259}}\right)$$

$$\frac{x-0.4514}{-0.5539} = \beta = \frac{y-2.6732}{0.8326}$$

$$\frac{x-0.4514}{-0.5539} = \frac{y-2.6732}{0.8326}$$

$$0.8326x - 0.3758 = -0.5539y + 1.4807$$
$$0.8326x + 0.5539y - 0.3758 - 1.4807 = 0$$
$$\boxed{0.8326x + 0.5539y - 1.8565 = 0}$$

Vector (En gráfica)

$$\vec{u} = P_0 - \vec{\mu}_1$$
$$= \begin{pmatrix} 0.4514 \\ 2.6732 \end{pmatrix} - \begin{pmatrix} \frac{1.503}{\sqrt{3.259}} \\ \frac{1}{\sqrt{3.259}} \end{pmatrix} = \begin{pmatrix} -0.3812 \\ 2.1193 \end{pmatrix}$$

$$\vec{v} = P_0 - \vec{\mu}_2$$
$$= \begin{pmatrix} 0.4514 \\ 2.6732 \end{pmatrix} - \begin{pmatrix} -\frac{1}{\sqrt{3.259}} \\ \frac{1.503}{\sqrt{3.259}} \end{pmatrix} = \begin{pmatrix} 1.0053 \\ 1.8406 \end{pmatrix}$$

- Ecuación de la cónica tranformada a su centro,

$$\tilde{p}^{t} A \tilde{p} + \tilde{\gamma} = 0$$

Teniendo $\tilde{\gamma} = -3.1753$

$$\tilde{\mathcal{C}} = \tilde{p}^{t} A \tilde{p} + \gamma = 0$$
$$= (\tilde{x}, \tilde{y}) \begin{pmatrix} 1.6 & \frac{1853}{200} \\ \frac{1853}{200} & -6.16 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} + (-3.1753)$$

$$\boxed{\tilde{\mathcal{C}} = 1.6\tilde{x}^2 + 18.53\tilde{x}\tilde{y} - 6.16\tilde{y} - 3.1753}$$

Cónica transformada al origen.

- Matrices de simetría

Siendo $\mu_1 = \begin{pmatrix} \frac{1.503}{\sqrt{3.259}} \\ \frac{1}{\sqrt{3.259}} \end{pmatrix}$

y $\mu_2 = \begin{pmatrix} -\frac{1}{\sqrt{3.259}} \\ \frac{1.503}{\sqrt{3.259}} \end{pmatrix}$

$$S_1 = \begin{pmatrix} \frac{2.259}{3.259} - \frac{1}{3.259} & \frac{1.503}{3.259} \\ \frac{1.503}{3.259} & \frac{1}{3.259} - \frac{2.259}{3.259} \end{pmatrix}$$

$$S_2 = \begin{pmatrix} \frac{1}{3.259} - \frac{2.259}{3.259} & -\frac{1.503}{3.259} \\ -\frac{1.503}{3.259} & \frac{2.259}{3.259} - \frac{1}{3.259} \end{pmatrix}$$

Guadarrama Rodríguez Vanesa

- Matriz Q tal que $\tilde{p} = Q\hat{p}$ transforme $\tilde{p}^t A \tilde{p} + \tilde{\gamma} = 0$ a $\hat{p}^t D \hat{p} + \gamma = 0$

$$\hat{p} = Q\hat{p}$$

$$(Q\hat{p})^t A (Q\hat{p}) + \tilde{\gamma} = 0$$
$$\hat{p}^t Q^t A Q \hat{p} + \tilde{\gamma} = 0$$
$$p^t D \hat{p} + \tilde{\gamma} = 0$$

con $D = Q^t A Q$
$QD = AQ$
$QDe_1 = AQe_1$
$Qe_1 = Aq_1$

ent. $D = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$Q = (u_1 \mid u_2)$

$\|u_1\| = \|u_2\| = 1$

$$\Rightarrow D = \begin{pmatrix} 7.7646 & 0 \\ 0 & -12.3246 \end{pmatrix}$$

$$y \quad Q = \begin{pmatrix} \frac{1.503}{\sqrt{3.259}} & -\frac{1}{\sqrt{3.259}} \\ \frac{1}{\sqrt{3.259}} & \frac{1.503}{\sqrt{3.259}} \end{pmatrix}$$

Calculando $p^t D \hat{p} + \hat{\gamma} = 0$

$$0 = (\hat{x}, \hat{y}) \begin{pmatrix} 7.7646 & 0 \\ 0 & -12.3246 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + (-3.1753)$$

$$0 = 7.7646\hat{x}^2 - 12.3246\hat{y}^2 - 3.1753$$

$$\boxed{3.1753 = 7.7646\hat{x}^2 - 12.3246\hat{y}^2}$$

Ecuación de la cónica en el origen y con sus ejes de simetría = eje "x" y eje "y"

- Tamaño de los ejes

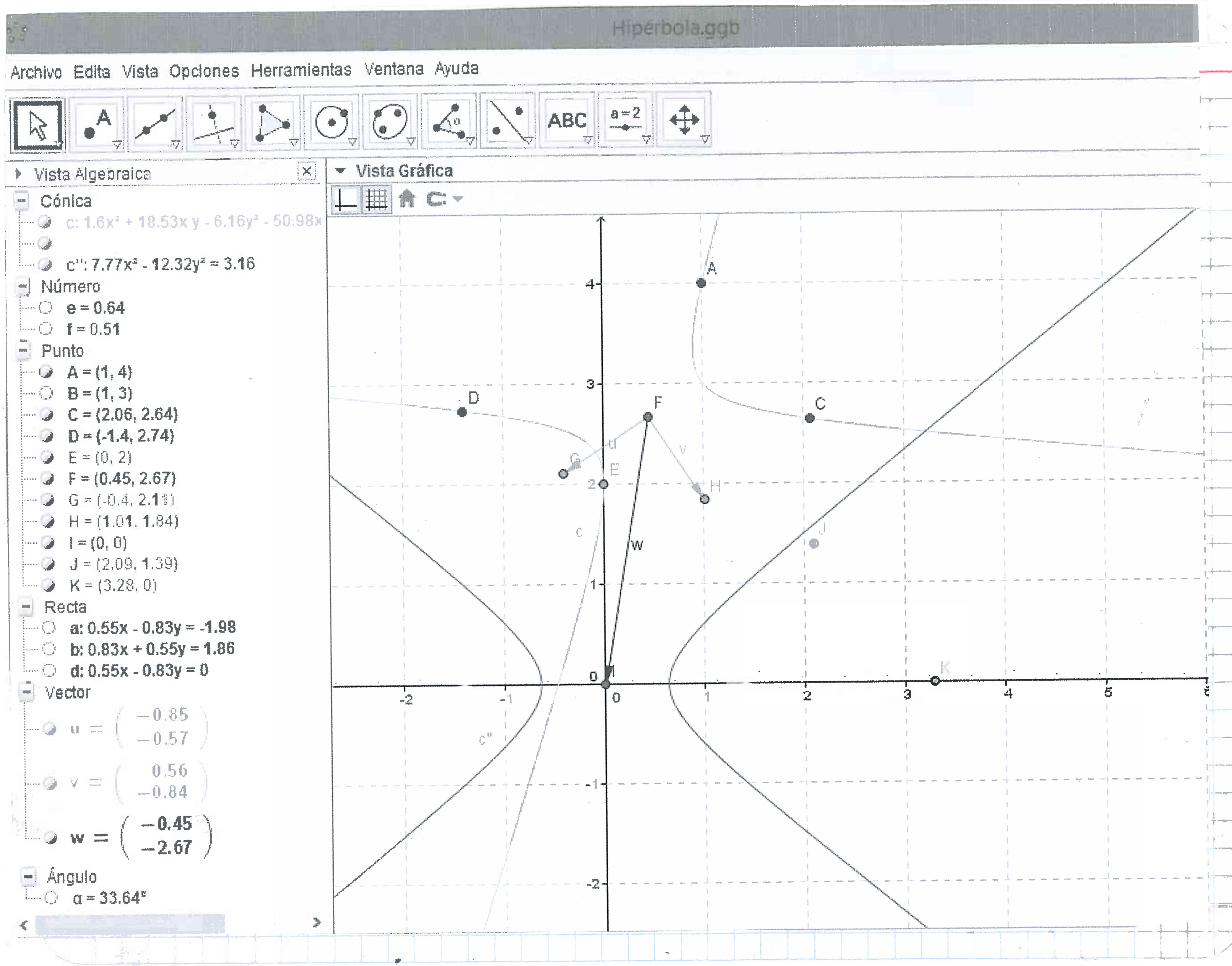
$$\frac{7.7646\hat{x}^2 - 12.3246\hat{y}^2 = 3.1753}{3.1753}$$

$$\frac{\hat{x}^2}{\frac{3.1753}{7.7646}} - \frac{y^2}{\frac{3.1753}{12.3246}} = 1$$

$\therefore \quad a = \sqrt{\frac{3.1753}{7.7646}} = 0.6395$

$b = \sqrt{\frac{3.1753}{12.3246}} = 0.5076$

$\therefore$ Semieje $a = 0.6395$ u
Semi eje $b = 0.5076$ u


Hipérbola.ggb
Archivo Edita Vista Opciones Herramientas Ventana Ayuda
ABC
a = 2
Vista Algebraica
Cónica
c: 1.6x² + 18.53x y - 6.16y² - 50.98x
c'': 7.77x² - 12.32y² = 3.16
Número
e = 0.64
f = 0.51
Punto
A = (1, 4)
B = (1, 3)
C = (2.06, 2.64)
D = (-1.4, 2.74)
E = (0, 2)
F = (0.45, 2.67)
G = (-0.4, 2.11)
H = (1.01, 1.84)
I = (0, 0)
J = (2.09, 1.39)
K = (3.28, 0)
Recta
a: 0.55x - 0.83y = -1.98
b: 0.83x + 0.55y = 1.86
d: 0.55x - 0.83y = 0
Vector
u = (-0.85, -0.57)
v = (0.56, -0.84)
w = (-0.45, -2.67)
Ángulo
α = 33.64°
Vista Gráfica
A
C
D
F
E
G
H
J
K
u
v
w
c
c''

Guadarrama Rodriguez Vanesa

Parabola: $9x^2-12xy+4y^2-44.84x-21.76y=-178.89$

$p^tAp+2b^tp+\gamma=0$

Sea $A=\begin{pmatrix}9 & -6\\ -6 & 4\end{pmatrix}$

$b=\begin{pmatrix}-\frac{1121}{50}\\ -\frac{272}{25}\end{pmatrix}$ y $\gamma=178.89$

Buscando P_0 = centro de la Parabola

$Ap_0=-b$

$$\begin{pmatrix}9 & -6\\ -6 & 4\end{pmatrix}\begin{pmatrix}x_0\\ y_0\end{pmatrix}=\begin{pmatrix}\frac{1121}{50}\\ \frac{272}{25}\end{pmatrix}$$

$$\left.\begin{array}{l}9x_0-6y_0=\frac{1121}{50}\\ -6x_0+4y_0=\frac{272}{25}\end{array}\right\}$$ No podemos hallar el P_0

No podemos eliminar términos lineales

* Calculando a $\lambda_{1,2}$

$A\mu=\lambda\mu$

Proponiendo $p=Q\hat{p}$

$(Q\hat{p})^tA(Q\hat{p})+2b^tQ\hat{p}+\gamma=0$

Usando $\det(A-\lambda I)=0$

$$\begin{vmatrix}9-\lambda & -6\\ -6 & 4-\lambda\end{vmatrix}=(\lambda-4)(\lambda-9)-36=0$$
$$=\lambda^2-13\lambda+36-36=0$$
$$=\lambda^2-13\lambda=0$$

Sea $\lambda_1=0$
y $\lambda_2=13$

* Para $\lambda_2=13$

$$\begin{cases}(9-\lambda)x+(-6)y=0 & (1)\\ -6x+(+4-\lambda)y=0 & (2)\end{cases}$$

Usando (1) y $\lambda_2=13$

$(9-13)x-6y=0$

$-4x-6y=0$

Sea $\vec{\mu}_1=\begin{pmatrix}-3\\ 2\end{pmatrix}$

y $\vec{\mu}_2=\begin{pmatrix}2\\ 3\end{pmatrix}$

Normalizando los vectores

$\|\mu\|=\sqrt{9+4}=\sqrt{13}$

$\therefore$ $\vec{\mu}_1=\begin{pmatrix}\frac{-3}{\sqrt{13}}\\ \frac{2}{\sqrt{13}}\end{pmatrix}$

y $\vec{\mu}_2=\begin{pmatrix}\frac{2}{\sqrt{13}}\\ \frac{3}{\sqrt{13}}\end{pmatrix}$

• Matrices de simetría

$$S_1=\begin{pmatrix}\frac{9}{13}-\frac{4}{13} & -\frac{12}{13}\\ -\frac{12}{13} & \frac{4}{13}-\frac{9}{13}\end{pmatrix}$$

$$S_2=\begin{pmatrix}\frac{4}{13}-\frac{9}{13} & \frac{12}{13}\\ \frac{12}{13} & \frac{9}{13}-\frac{4}{13}\end{pmatrix}$$

• Buscar D y Q

Como $\tilde{p}=Q\hat{p}$

$(Q\hat{p})^tA(Q\hat{p})+2b^t(Q\hat{p})+\gamma=0$

$\hat{p}^tD\hat{p}+2b^tQ\hat{p}+\gamma=0$

Sea $Q=\frac{1}{\sqrt{13}}\begin{pmatrix}-3 & 2\\ 2 & 3\end{pmatrix}$

y $D=\begin{pmatrix}13 & 0\\ 0 & 0\end{pmatrix}$

Calculando $b^t Q$

$$b^t Q \frac{1}{\sqrt{13}}\begin{pmatrix} -\frac{1121}{50} \\ -\frac{272}{25} \end{pmatrix}\begin{pmatrix} -3 & 2 \\ 2 & 3 \end{pmatrix}$$

$$= \frac{1}{\sqrt{13}}\begin{pmatrix} \frac{91}{2} & -\frac{1937}{25} \end{pmatrix}$$

Calculando $\hat{p}^t D \hat{p} + 2b^t Q \hat{p} + \gamma = 0$

$$(\hat{x}, \hat{y})\begin{pmatrix} 13 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + \frac{2}{\sqrt{13}}\begin{pmatrix} \frac{91}{2} & -\frac{1937}{25} \end{pmatrix}\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + 178.89 = 0$$

$$13\hat{x}^2 + \left(\frac{2}{\sqrt{13}}\right)\left(\frac{91}{2}\right)\hat{x} - \left(\frac{2}{\sqrt{13}}\right)\left(\frac{1937}{25}\right)\hat{y} + 178.89 = 0$$

$$13\hat{x}^2 + 7\sqrt{13}\,\hat{x} - 42.97822\hat{y} + 178.89 = 0$$

$$13\left(\hat{x}^2 + \frac{7\sqrt{13}}{13}\hat{x} + \left(\frac{7\sqrt{13}}{26}\right)^2\right) - 42.9782\hat{y} + 178.89 - \frac{49}{4} = 0$$

$$\frac{13\left(\hat{x}^2 + \frac{7\sqrt{13}}{26}\right)^2}{13} = \frac{42.9782\hat{y} + 166.64}{13}$$

$$\left(\hat{x}^2 + \frac{7\sqrt{13}}{26}\right)^2 = 3.3060\hat{y} + 12.8185$$

$$\boxed{\left(\hat{x}^2 + \frac{7\sqrt{13}}{26}\right)^2 = 3.306\left(\hat{y} + 3.8773\right)}$$ Ec. de la cónica rotada

y podemos obtener $P_0 = \left(-\frac{7\sqrt{13}}{26}, -3.8773\right)$

Calcular la ecuación de la cónica con centro en $(0,0)$

$$(\tilde{x} - 0)^2 = 3.306(\tilde{y} - 0)$$

$$\boxed{\tilde{x}^2 = 3.306\tilde{y}}$$ ecuación de la cónica en el origen y rotada

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