Geometría Analítica II

Trabajo 25

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Hacer los ejercios de la página 69, del libro: Analytic Geometry of Space, de Virgil Snyder, el libro que hemos seguido de cerca. Se anexa la hoja de los ejercicios.

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EXERCISES

1. By translating the axes of coördinates, show that the surface defined by the equation $2x^2 + 3y^2 + 4z^2 - 4x - 6y + 16z + 16 = 0$ is an ellipsoid. Find the coördinates of the center and the lengths of the semi-axes.

2. Classify and describe the surface $x^2 + y^2 - 4x - 3y + 10z = 20 - z^2$.

3. Show that the surface $2x^2 - 3z^2 - 5z = 7 - 2y^2$ is a surface of revolution. Find the equations of the generating curve.

4. On the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$, find the equations of the two lines which pass through the point (1, 0, 0); through (– 1, 0, 0).

5. Classify and plot the loci defined by the following equations:

(*a*) $9x^2 + 16y^2 + 25z^2 = 1$,
(*b*) $4x^2 - 9y^2 - 16z^2 = 25$,
(*c*) $4x^2 - 16y^2 + 9z^2 = 25$,
(*d*) $x^2 + y^2 - 4z^2 = 25$,
(*e*) $x^2 + 4y^2 + z^2 = 9$,
(*f*) $x^2 + 4y^2 + 9z^2 + 8 = 0$.

60. The elliptic paraboloid. The locus of the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2nz$$

is called an **elliptic paraboloid**. The surface is symmetric as to the planes $x = 0$ and $y = 0$ but not as to $z = 0$. It passes through the origin, and lies on the positive side of $z = 0$ if n is positive and on the negative side if n is negative. In the following discussion it will be assumed that n is positive. If n is negative, it is necessary only to reflect the surface on the plane $z = 0$.

The section of the paraboloid by the plane $z = k$ is an ellipse whose semi-axes are $a\sqrt{2nk}$ and $b\sqrt{2nk}$, respectively. If $k < 0$, the ellipse is imaginary. If $k = 0$, the ellipse reduces to a point, the origin. As k increases, the semi-axes of the ellipse increase without limit.

The section of the paraboloid by the plane $y = k'$ is the parabola

$$\frac{x^2}{a^2} = 2nz - \frac{k'^2}{b^2}, \quad y = k'.$$

For all values of k' these parabolas are congruent. As k' increases, the vertices recede from the plane $y = 0$ along the parabola

$$\frac{y^2}{b^2} = 2nz, \quad x = 0.$$