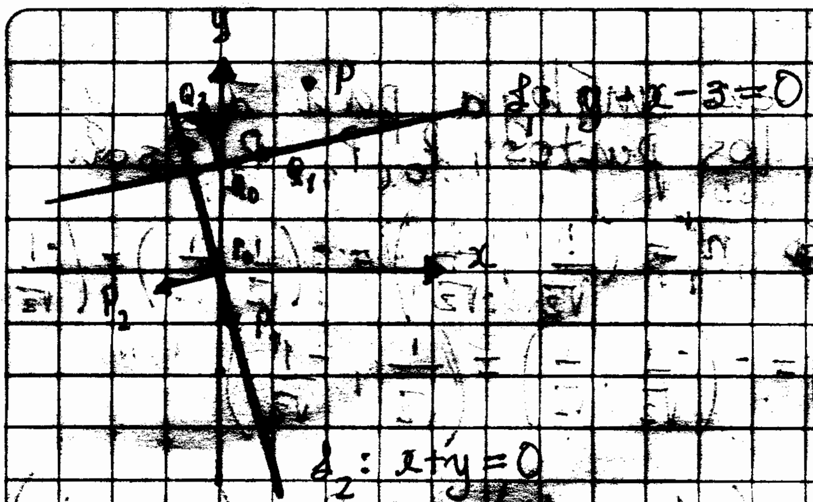
MARTINEZ JIMENEZ MANUEL GEOMETRIA ANALITICA




Primero obtengo los puntos $Q_0, Q_1, Q_2$, fijándome que el vector normal $\vec{n}$ de $\ell_1$ es $\vec{n} = (-1,+1)$, lo normalizo y obtengo: $\vec{n}_1 = \left(\frac{-1}{\sqrt{2}}, \frac{+1}{\sqrt{2}}\right)$ entonces el vector dirección normalizado de $\ell_1$ es $\left(\frac{+1}{\sqrt{2}}, \frac{+1}{\sqrt{2}}\right)$ ó $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

Ahora obtengo primero el punto $Q_0$: si $x=0$ en $\ell_1$ entonces obviamente $y=3$ entonces $Q_0=(0,3)$ para los demás $Q_1, Q_2$ sólo sumo las componentes de $\vec{n}_1$ y $\vec{d}_1$ a $Q_0$, luego $Q_1 = \left(\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$ $Q_2 = \left(\frac{-1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

entonces:

$$Q_0 = (0,3)$$

$$Q_1 = \left(\frac{1}{\sqrt{2}}, \frac{3+1}{\sqrt{2}}\right)$$

$$Q_2 = \left(\frac{-1}{\sqrt{2}}, \frac{3+1}{\sqrt{2}}\right)$$

El procedimiento ~~es~~ análoga para $\mathcal{L}_2$ y así obtengo que los puntos $P_0, P_1, P_2$ son

$$\mathcal{L}_2: \quad \vec{n} = (1,1) \Rightarrow \vec{n}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$\vec{d}_1 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$P_0 = (0,0) \Rightarrow P_1 = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \Rightarrow P_2 = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

Entonces:

$$P_0 = (0,0) \qquad \vec{P_0P_1} = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$P_1 = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \Rightarrow \vec{P_0P_2} = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$P_2 = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

AHORA HAGO LAS TRASFORMACIONES DE COORDENADAS

$$Q_0 = (0,3)$$

$$Q_1 = \left(\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right) \Rightarrow \vec{Q_0Q_1} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$Q_2 = \left(\frac{-1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right) \qquad \vec{Q_0Q_2} = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\vec{OP} = \vec{OQ_0} + \vec{Q_0P}$$

$$(x,y) = (0,3) + \alpha\,(\vec{Q_0Q_1}) + \beta\,(\vec{Q_0Q_2})$$

$$(x,y) = (0,3) + \alpha\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) + \beta\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x = \frac{\alpha}{\sqrt{2}} + \frac{-\beta}{\sqrt{2}} + 0 \;;\quad y = \frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}} + 3$$

EN FORMA MATRICIAL:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

PARA LOS $P_0, P_1, P_2$ ANALOGO

$$\vec{OP} = \vec{OP_0} + \vec{P_0P}$$

$$(x,y) = (0,0) + \bar{\alpha}\,(\vec{P_0P_1}) + \bar{\beta}\,(\vec{P_0P_2})$$

$$(x,y) = \bar{\alpha}\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) + \bar{\beta}\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$(x,y) = \left(\frac{\bar{\alpha}}{\sqrt{2}}, -\frac{\bar{\alpha}}{\sqrt{2}}\right) + \left(\frac{-\bar{\beta}}{\sqrt{2}}, -\frac{\bar{\beta}}{2}\right)$$

entonces: $x = \frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}$

$y = \frac{-1}{\sqrt{3}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}$

EN FORMA MATRICIAL:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

PARA $\bar{\alpha}$ y $\bar{\beta}$ tengo que:

$$x = \frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}$$

$$y = -\frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}$$

$$x + y = -\frac{2}{\sqrt{2}}\beta = -\sqrt{2}\beta \Rightarrow \bar{\beta} = -\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y$$

Para $\bar{\alpha}$

$$-x = \frac{-1}{\sqrt{2}}\bar{\alpha} + \frac{1}{\sqrt{2}}\bar{\beta}$$

$$y = -\frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\beta$$

$$y - x = -\frac{2}{\sqrt{2}}\alpha = -\sqrt{2}\,\bar{\alpha} \Rightarrow \bar{\alpha} = -\frac{1}{\sqrt{2}}y + \frac{1}{\sqrt{2}}x$$

Por lo tanto:

$$\bar{\alpha} = -\frac{1}{\sqrt{2}}y + \frac{1}{\sqrt{2}}x$$

$$\bar{\beta} = -\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y$$

Para $\alpha$ y $\beta$ tengo:

$$x = \frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta$$

$$y = \frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta + 3$$

$$x + y = \frac{2}{\sqrt{2}}\alpha + 3$$

$$x + y - 3 = \sqrt{2}\,\alpha \Rightarrow \alpha = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{3}{\sqrt{2}}$$

En otro caso:

$$-x = -\frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta$$

$$y = \frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta + 3$$

$$y - x = \frac{2}{\sqrt{2}}\beta + 3 = \sqrt{2}\,\beta + 3$$

$$y - x - 3 = \sqrt{2}\,\beta \Rightarrow \beta = \frac{1}{\sqrt{2}}y - \frac{1}{\sqrt{2}}x - \frac{3}{\sqrt{2}}$$

Entonces:

$$\alpha = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{3}{\sqrt{2}} \quad \blacksquare$$

$$\beta = \frac{-1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{3}{\sqrt{2}} \quad \blacksquare$$

Por otra parte

$$\alpha = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}\right) + \frac{1}{\sqrt{2}}\left(\frac{-1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}\right) - \frac{3}{\sqrt{2}}$$

$$\alpha = \frac{1}{2}\bar{\alpha} - \frac{1}{2}\bar{\beta} - \frac{1}{2}\bar{\alpha} - \frac{1}{2}\bar{\beta} - \frac{3}{\sqrt{2}}$$

$$\alpha = -\bar{\beta} - \frac{3}{\sqrt{2}} \quad \blacksquare$$

$$\beta = \frac{-1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}\right) + \frac{1}{\sqrt{2}}\left(\frac{-1}{\sqrt{2}}\bar{\alpha} - \frac{1}{\sqrt{2}}\bar{\beta}\right) - \frac{3}{\sqrt{2}}$$

$$\beta = \frac{-1}{2}\bar{\alpha} + \frac{1}{2}\bar{\beta} - \frac{1}{2}\bar{\alpha} - \frac{1}{2}\bar{\beta} - \frac{3}{\sqrt{2}}$$

$$\beta = -\bar{\alpha} - \frac{3}{\sqrt{2}} \quad \blacksquare$$

EN NOTACION MATRICIAL

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \bar{\alpha} \\ \bar{\beta} \end{pmatrix} + \begin{pmatrix} -3/\sqrt{2} \\ -3/\sqrt{2} \end{pmatrix}$$

SIGUE ATRÁS:

En tanto:

$$\overline{\alpha} = -\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta\right) + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta\right)$$

$$\overline{\alpha} = -\frac{1}{2}\alpha - \frac{1}{2}\beta + \frac{3}{\sqrt{2}} + \frac{1}{2}\alpha - \frac{1}{2}\beta$$

$$\overline{\alpha} = -\beta + \frac{3}{\sqrt{2}}$$

$$\overline{\beta} = -\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta + 3\right)$$

$$\overline{\beta} = \left(-\frac{1}{2}\alpha + \frac{1}{2}\beta - \frac{1}{2}\alpha - \frac{1}{2}\beta - \frac{3}{\sqrt{2}}\right)$$

$$\overline{\beta} = -\alpha - \frac{3}{\sqrt{2}}$$

EN NOTACIÓN MATRICIAL

$$\begin{pmatrix} \overline{\alpha} \\ \overline{\beta} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} -3/\sqrt{2} \\ -3/\sqrt{2} \end{pmatrix}$$