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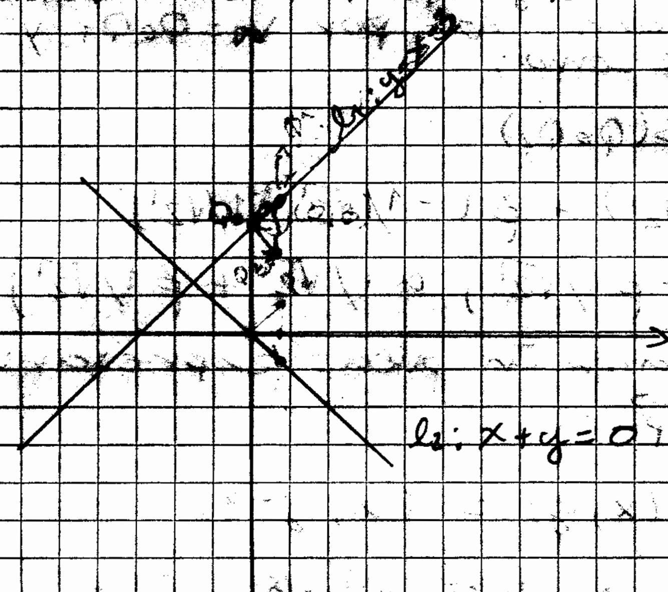
Geometría Analítica II

Considérense las rectas:



⇝ Para $l_1: y + x - 3 = 0$,

Tomar $Q_1(x_1, y_1)$, tal que $Q_1 \in l_1$, y

$\|Q_0Q_1\| = 1 = x_1^2 - (y_1 - 3)^2$, i.e

$1 = x_1^2 - (\cancel{3} + x_1 - \cancel{3})^2$

$1 = x_1^2 + x_1^2$

$x_1 = \dfrac{1}{\sqrt{2}}$

$\Rightarrow y_1 = 3 + 1/\sqrt{2}$

$Q_1Q_0 = (1/\sqrt{2}, 3 + 1/\sqrt{2}) - (0, 3) = (1/\sqrt{2}, 1/\sqrt{2})$

⇝ Encontrar $\overrightarrow{Q_0Q_2}$ tal que $\overrightarrow{Q_0Q_2} \perp \overrightarrow{Q_0Q_1}$

y $\|Q_0Q_2\| = 1$

$\Rightarrow Q_0Q_2 = (1/\sqrt{2}, -1/\sqrt{2})$

$Q_2 - Q_0 = (1/\sqrt{2}, -1/\sqrt{2}) = (x_2, y_2) - (0, 3)$

$1/\sqrt{2} = x_2$

$-1/\sqrt{2} + 3 = y_2$

$\Rightarrow Q_2 = (1/\sqrt{2}, -1/\sqrt{2} + 3)$

Luego, considerar $P(x,y)$. Q_0P, en el sistema de referencia formado por $V_1 = Q_0Q_1$ y $V_2 = Q_0Q_2$, puede expresarse como:

$$\overline{Q_0P} = \alpha\, Q_0Q_1 + \beta(Q_0Q_2)$$
$$= \alpha\,(1/\sqrt{2}, 1/\sqrt{2}) + \beta(-1/\sqrt{2}, 1/\sqrt{2})$$
$$= (\alpha\, 1/\sqrt{2} - \beta\, 1/\sqrt{2},\ \alpha\, 1/\sqrt{2} + \beta\, 1/\sqrt{2})$$

Pero el $\overrightarrow{Q_0P}$, también puede expresarse

$$\overrightarrow{Q_0P} = \overrightarrow{Q_0O} + \overrightarrow{OP}$$
$$= (0,-3) + (x,y) = (x, y-3)$$

$$\Rightarrow \quad (x, y-3) = \left(\frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta,\ \frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta\right)$$

igualando componentes:

(i) $$x = \frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta$$

$$y = \frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta + 3$$

en forma matricial:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

ii) Por otra parte:

Considerar $P_0(0,0) \in l_2$; se busca $P_1 \in l_2$ tal que $\|P_0P_1\| = 1$

$$1 = x_1^2 + y_1^2$$
$$1 = x_1^2 + (-x_1)^2$$
$$\frac{1}{\sqrt{2}} = x_1$$

$$\Rightarrow P_1(1/\sqrt{2}, -1/\sqrt{2}) \quad \text{y} \quad P_0P_1 = (1/\sqrt{2}, -1/\sqrt{2})$$

Luego, se busca P_2, tal que, $P_0P_2 \perp P_0P_1$ y $\|P_0P_2\| = 1$

Si se hace, $P_0P_2 = (1/\sqrt{2}, 1/\sqrt{2})$, se satisfacen las condiciones anteriores.

$\Rightarrow P_0P_2 = P_2 - P_0 = (1/\sqrt{2}, 1/\sqrt{2}) = (x_2, y_2) - (0,0)$

$$x_2 = 1/\sqrt{2} \quad ; \quad y_2 = 1/\sqrt{2}$$

$P_2 = (1/\sqrt{2}, 1/\sqrt{2})$

Sea $P(x,y)$. P_0P, en el sistema de referencia formado por $\{P_0P_1, P_0P_2\}$, puede expresarse como combinación lineal de estos vectores.

$$\begin{aligned}\overrightarrow{P_0P} &= \tilde{\alpha}\, P_0P_1 + \tilde{\beta}\, P_0P_2 \\ &= \tilde{\alpha}(1/\sqrt{2}, -1/\sqrt{2}) + \tilde{\beta}(1/\sqrt{2}, 1/\sqrt{2}) \\ &= \left(\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}, -\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}\right)\end{aligned}$$

Pero el $\overrightarrow{P_0P}$, puede verse como

$$\begin{aligned}\overrightarrow{P_0P} &= \overrightarrow{P_0O} + \overrightarrow{OP} \\ &= (0,0) + (x,y) \\ &= (x,y)\end{aligned}$$

$$\Rightarrow (x,y) = \left(\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}, -\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}\right)$$

igualando componentes

(i.e) $x = \frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}$

$y = -\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}$

en forma matricial:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{pmatrix}$$

De (i) y (ii)

$$\frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta = \frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}$$

$$\frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta + 3 = -\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}$$

Resolviendo el sistema para α y β:

$$\frac{1}{\sqrt{2}}\alpha = \frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta} + \frac{1}{\sqrt{2}}\beta$$

$$\alpha = \tilde{\alpha} + \tilde{\beta} + \beta$$

$$\frac{1}{\sqrt{2}}(\tilde{\alpha} + \tilde{\beta} + \beta) + \frac{1}{\sqrt{2}}\beta + 3 = -\frac{1}{\sqrt{2}}\tilde{\alpha} + \frac{1}{\sqrt{2}}\tilde{\beta}$$

$$\frac{1}{\sqrt{2}}\tilde{\alpha} + \cancel{\frac{1}{\sqrt{2}}\tilde{\beta}} + \frac{1}{\sqrt{2}}\beta + \frac{1}{\sqrt{2}}\beta + 3 = -\frac{1}{\sqrt{2}}\tilde{\alpha} + \cancel{\frac{1}{\sqrt{2}}\tilde{\beta}}$$

$$\frac{2}{\sqrt{2}}\beta = -\frac{2}{\sqrt{2}}\tilde{\alpha} - 3$$

$$\beta = -\tilde{\alpha} - \frac{3\sqrt{2}}{2}$$

$$\Rightarrow \alpha = \cancel{\tilde{\alpha}} + \tilde{\beta} + \left(\cancel{-\tilde{\alpha}} - \frac{3\sqrt{2}}{2}\right)$$

$$\alpha = \tilde{\beta} - \frac{3\sqrt{2}}{2}$$

i.e. $\alpha = \tilde{\beta} - \frac{3\sqrt{2}}{2}$

$$\beta = -\tilde{\alpha} - \frac{3\sqrt{2}}{2}$$

en forma matricial:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{pmatrix} + \begin{pmatrix} -3\sqrt{2}/2 \\ -3\sqrt{2}/2 \end{pmatrix}$$