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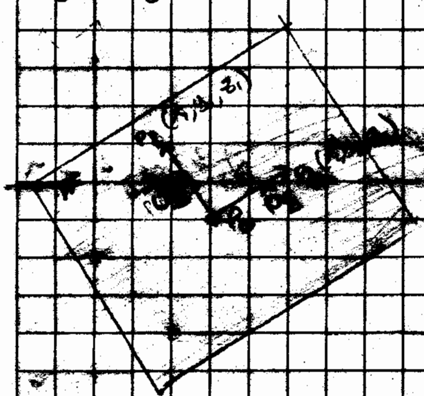
Geometría Analítica II

Identificar a la circunferencia que vive en el plano $\pi = \{(x,y,z) \mid x+y+z=1\}$

Cuyo origen es $P_0(1,1,-1)$ y tiene radio $r=1$



Sea $P_1(x_1,y_1,z_1)$ y $P_1 \in \pi \Rightarrow x_1+y_1+z_1=1 \Rightarrow$

$(x_1-1)+(y_1-1)+(z_1+1)=0$

Si $z_1, y_1=0 \Rightarrow x_1-1-1+1=0 \Rightarrow x_1=1$

$\Rightarrow P_1(1,0,0)$

Sea $P_2(x_2,y_2,z_2)$ y $P_2 \in \pi \Rightarrow \boxed{x_2+y_2+z_2=1}$ ①

Queremos que $(x_2-1, y_2-1, z_2+1) \perp (0,-1,1)$ i.e.

$(x_2-1, y_2-1, z_2+1)\cdot(0,-1,1)=0 \Rightarrow (y_2-1)(-1)+z_2+1=0 \Rightarrow -y_2+1+z_2+1=0 \Rightarrow$

$\Rightarrow -y_2+z_2+2=0 \Rightarrow \boxed{y_2-z_2-2=0}$ ②

restando ② de ① $\Rightarrow x_2+2z_2+1=0$

Si hacemos $z_2=0 \Rightarrow \boxed{x_2=-1}$ y $\boxed{y_2=2} \Rightarrow$

$P_2(-1,2,0)$

$\vec{P_0P_1}=(x_1-1, y_1-1, z_1+1) \Rightarrow (1-1, 0-1, 0+1) = (0,-1,1) = \vec{u}_1$

$\vec{P_0P_2}=(x_2-1, y_2-1, z_2+1) = (-1-1, 2-1, 0+1) = (-2,1,1) = \vec{u}_2$

Comprobando

$\vec{u}_1 \cdot \vec{u}_2 = (0,-1,1)\cdot(-2,1,1) = -1+1 = 0 \therefore \vec{u}_1 \perp \vec{u}_2$

normalizando $\vec{u}_1$ y $\vec{u}_2$

$\hat{u}_1 = \frac{\vec{u}_1}{\|\vec{u}_1\|} = \frac{1}{\sqrt{2}}(0,-1,1) = (0, -1/\sqrt{2}, 1/\sqrt{2})$

$\hat{u}_2 = \frac{\vec{u}_2}{\|\vec{u}_2\|} = \frac{1}{\sqrt{6}}(-2,1,1) = (-2/\sqrt{6}, 1/\sqrt{6}, 1/\sqrt{6})$

nuestro nuevo sistema ortonormal de referencia será =

$\bar{P} = P_0 + d_1\hat{u}_1 + d_2\hat{u}_2$ para $\bar{P}(x,y,z)$

tenemos pues:

$(x,y,z) = (1,1,-1) + \alpha_1(0, -1/\sqrt{2}, 1/\sqrt{2}) + \alpha_2(-2/\sqrt{6}, 1/\sqrt{6}, 1/\sqrt{6})$

reescribiendo:

$$\left.\begin{aligned} x &= 1 - 2/\sqrt{6}\,\alpha_2 \\ y &= 1 - 1/\sqrt{2}\,\alpha_1 + 1/\sqrt{6}\,\alpha_2 \\ z &= -1 + 1/\sqrt{2}\,\alpha_1 + 1/\sqrt{6}\,\alpha_2 \end{aligned}\right\} \Rightarrow$$

la ecuación implícita de la circunferencia es $\alpha_1^2 + \alpha_2^2 = 1$

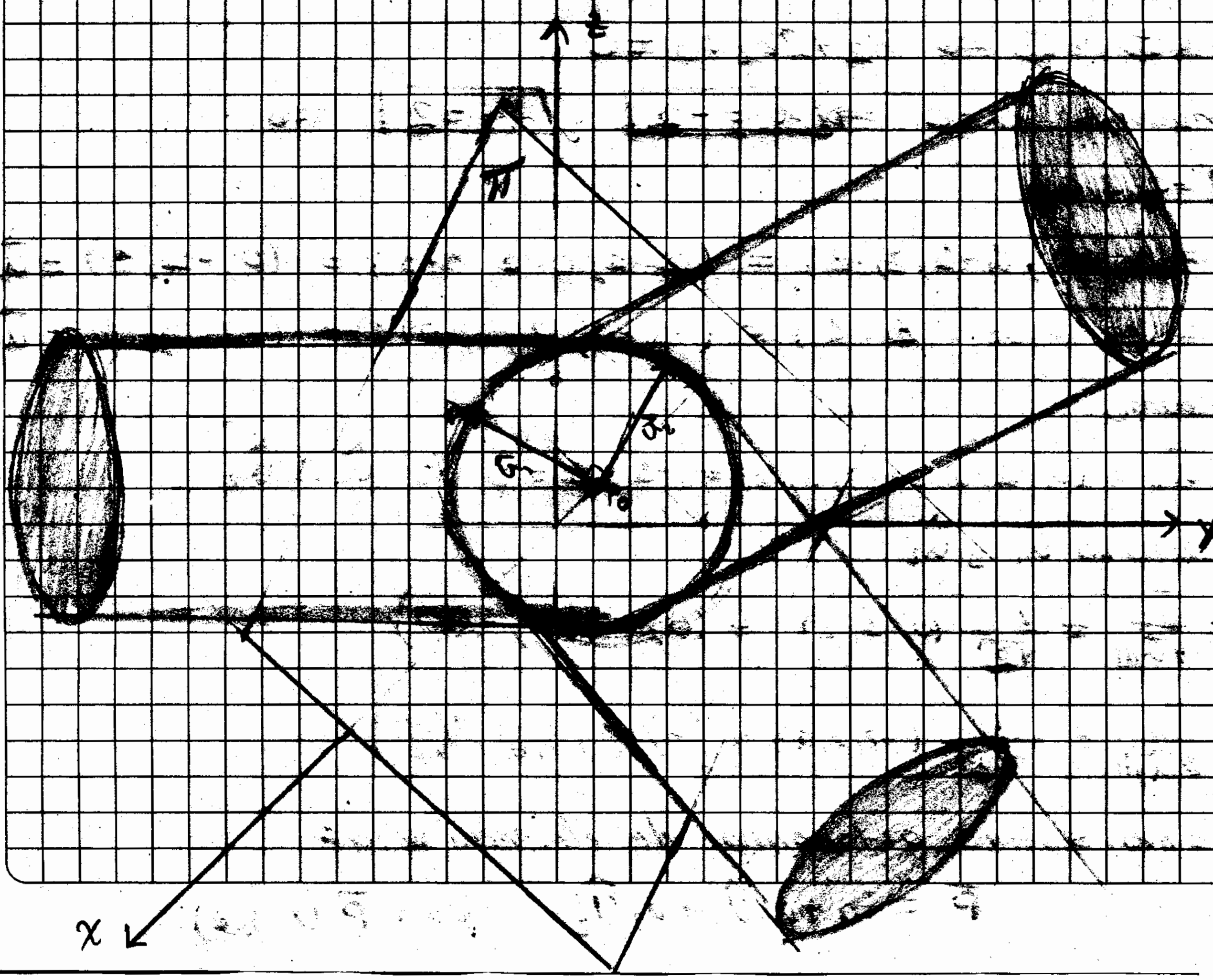
Para la ec. explícita en $\mathbb{R}^2$ es

$$x = r\cos\theta$$
$$y = r\,\mathrm{sen}\,\theta$$

Sabiendo que $r = 1$ tenemos

(I)
$$\boxed{\begin{aligned} x &= 1 - 2/\sqrt{6}\,\mathrm{sen}\,\theta \\ y &= 1 - 1/\sqrt{2}\cos\theta + 1/\sqrt{6}\,\mathrm{sen}\,\theta \\ z &= -1 + 1/\sqrt{2}\cos\theta + 1/\sqrt{6}\,\mathrm{sen}\,\theta \end{aligned}}$$

Ecuación explícita de la circunferencia en $\mathbb{R}^3$



tenemos x & y eliminando z de (i)

$$\left.\begin{array}{l} x-1=-\tfrac{2}{\sqrt{6}}\operatorname{sen}\theta \\ y-1=-\tfrac{1}{\sqrt{2}}\cos\theta+\tfrac{1}{\sqrt{6}}\operatorname{sen}\theta \end{array}\right\} \Rightarrow \operatorname{sen}\theta=-\tfrac{\sqrt{6}}{2}(x-1)$$

$$y-1=-\tfrac{1}{\sqrt{2}}\cos\theta+\tfrac{1}{\sqrt{6}}\left(-\tfrac{\sqrt{6}}{2}(x-1)\right) \Rightarrow -\tfrac{1}{\sqrt{2}}\cos\theta=(y-1)+\tfrac{1}{2}(x-1) \Rightarrow$$

$$\Rightarrow \cos\theta=-\sqrt{2}\left[(y-1)+\tfrac{1}{2}(x-1)\right]$$

pero sabemos que $\operatorname{sen}^2\theta+\cos^2\theta=1 \Rightarrow$

$$\left[-\tfrac{\sqrt{6}}{2}(x-1)\right]^2+\left[-\sqrt{2}\left[(y-1)+\tfrac{1}{2}(x-1)\right]\right]^2=1$$

$$\tfrac{6}{4}(x-1)^2+2\left[(y-1)+(\tfrac{1}{2}x-\tfrac{1}{2})\right]^2=1 \Rightarrow$$

$$\tfrac{3}{2}(x-1)^2+2\left[(y-1)^2+2(y-1)(\tfrac{1}{2}x-\tfrac{1}{2})+(\tfrac{1}{2}x-\tfrac{1}{2})^2\right]=1 \Rightarrow$$

$$\Rightarrow \tfrac{6}{4}x^2-3x+\tfrac{6}{4}+2\left[y^2-2y+1+xy-y-x+1+\tfrac{1}{4}x^2-\tfrac{1}{2}x+\tfrac{1}{4}\right]=1 \Rightarrow$$

$$\Rightarrow \tfrac{3}{2}x^2-3x+\tfrac{3}{2}+2y^2-4y+2+2xy-2y-2x+2+\tfrac{1}{2}x^2-x+\tfrac{1}{2}=1 \Rightarrow$$

$$\Rightarrow 2x^2-8x+2y^2-6y+6+2xy=1 \Rightarrow$$

$$\Rightarrow 2(x^2-4x+y^2-3y+3+xy)=1 \Rightarrow 2(x^2-2x+y^2-2y+1+1+1+xy-2x-y)=1 \Rightarrow$$

$$\Rightarrow 2(x^2-2x+1+y^2-2y+1+xy-2x-y)=2 \Rightarrow 2((x-1)^2+(y-1)^2+(xy-2x-y))=1$$

$$\Rightarrow 2(x-1)^2+2(y-1)^2+2(xy-2x-y)=1$$

Esto se puede escribir como =

$$\boxed{2\tilde{x}^2+2\tilde{y}^2+2\tilde{x}\tilde{y}=1}$$ Es la ecuación de una elipse en el plano xy

Eliminemos x de (i) entonces tenemos =

$$\left.\begin{array}{l} y=1-\tfrac{1}{\sqrt{2}}\cos\theta+\tfrac{1}{\sqrt{6}}\operatorname{sen}\theta \\ z=-1+\tfrac{1}{\sqrt{2}}\cos\theta+\tfrac{1}{\sqrt{6}}\operatorname{sen}\theta \end{array}\right\} \Rightarrow$$

que quedara $2\tilde{y}^2+2\tilde{z}^2+2\tilde{y}\tilde{z}=1$ es la ec de la elipse en el plano yz

y Analogamente eliminando y de (i) tenemos

$$\begin{array}{l} x-1=-\tfrac{2}{\sqrt{6}}\operatorname{sen}\theta \\ z+1=\tfrac{1}{\sqrt{2}}\cos\theta+\tfrac{1}{\sqrt{6}}\operatorname{sen}\theta \end{array}$$ quedando $2\tilde{x}^2+2\tilde{z}^2+2\tilde{x}\tilde{z}=0$

Es la ec. de la elipse en el plano xz.