Patricia A. Vélez Mellado 7/mar/05

Geometría Analítica II

→ Considerar los ptos. $P_0(1,1,2)$; $P_1(1,-1,2)$; $P_2(1,2,3)$ y $P_3(3,2,1)$ cambiar del sistema de referencia $\{P_0, P_1, P_2, P_3\}$ al formado por $Q_0(3,0,-1)$; $Q_1(4,2,2)$; $Q_2(2,2,4)$ $Q_3(0,1,-1)$.

Considerar el pto. $P(x,y,z)$, el $\overrightarrow{P_0P}$ se puede expresar como:

$$\overrightarrow{P_0P} = \alpha_1 \overrightarrow{P_0P_1} + \alpha_2 \overrightarrow{P_0P_2} + \alpha_3 \overrightarrow{P_0P_3}$$

luego, el $\overrightarrow{OP}$,

$$\overrightarrow{OP} = \overrightarrow{OP_0} + \overrightarrow{P_0P}$$

i.e.

$$\overrightarrow{OP} = \overrightarrow{OP_0} + \alpha_1 \overrightarrow{P_0P_1} + \alpha_2 \overrightarrow{P_0P_2} + \alpha_3 \overrightarrow{P_0P_3}$$

con $\overrightarrow{P_0P_1} = (0,-2,0)$; $\overrightarrow{P_0P_2} = (0,1,1)$,

$\overrightarrow{P_0P_3} = (2,1,-1)$

$$\Rightarrow \overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

igualando componentes:

$x = 1 + 2\alpha_3$;

$y = 1 - 2\alpha_1 + \alpha_2 + \alpha_3$

$z = 2 + \alpha_2 - \alpha_3$

en forma matricial:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ -2 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

Para Q_0, Q_1, Q_2, Q_3; se tiene que Q_0P se puede expresar como:

$$\overrightarrow{Q_0P} = \beta_1 \overrightarrow{Q_0Q_1} + \beta_2 \overrightarrow{Q_0Q_2} + \beta_3 \overrightarrow{Q_0Q_3}$$

luego el pto. $\overrightarrow{OP}$,

$$\overrightarrow{OP} = \overrightarrow{OQ_0} + \overrightarrow{Q_0P}$$

i.e. $\overrightarrow{OP} = \overrightarrow{OQ_0} + \beta_1 \overrightarrow{Q_0Q_1} + \beta_2 \overrightarrow{Q_0Q_2} + \beta_3 \overrightarrow{Q_0Q_3}$

con $Q_0Q_1 = (1,2,3)$; $\overrightarrow{Q_0Q_2} = (-1,2,5)$; $\overrightarrow{Q_0Q_3} = (-3,1,0)$

$$\Rightarrow \overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \beta_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \beta_2 \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + \beta_3 \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$$

igualando componentes:

$$\begin{aligned} x &= 3 + \beta_1 - \beta_2 - 3\beta_3 \\ y &= 2\beta_1 + 2\beta_2 + \beta_3 \\ z &= -1 + 3\beta_1 + 5\beta_2 \end{aligned}$$

en forma matricial:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & -3 \\ 2 & 2 & 1 \\ 3 & 5 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \quad \cdots \quad (i)$$

$\rightsquigarrow$ En el sistema de referencia P_0, P_1, P_2, P_3:

$$\alpha_3 = \frac{x}{2} - \frac{1}{2}$$

$$\alpha_2 = z - 2 + \alpha_3 = z - 2 + \frac{x}{2} - \frac{1}{2} = z + \frac{1}{2}x - \frac{5}{2}$$

$$\alpha_1 = \frac{1}{2} - \frac{y}{2} + \frac{\alpha_2}{2} + \frac{\alpha_3}{2}$$

$$= \frac{1}{2} - \frac{1}{2}y + \frac{1}{2}z + \frac{1}{4}x - \frac{5}{4} + \frac{x}{4} - \frac{1}{4}$$

$$= \frac{1}{2}x - \frac{1}{2}y + \frac{1}{2}z - 1$$

$$\Rightarrow \alpha_1 = \frac{1}{2}x - \frac{1}{2}y + \frac{1}{2}z - 1$$

$$\alpha_2 = z + \frac{1}{2}x - \frac{5}{2}$$

$$\alpha_3 = \frac{x}{2} - \frac{1}{2}$$

Sust. de (i):

$$\Rightarrow \alpha_1 = \frac{1}{2}(3+\beta_1-\beta_2-3\beta_3) - \frac{1}{2}(2\beta_1+2\beta_2+\beta_3)$$

$$+ \frac{1}{2}(-1+3\beta_1+5\beta_2) - 1$$

$$= \frac{3}{2} + \frac{1}{2}\beta_1 - \frac{1}{2}\beta_2 - \frac{3}{2}\beta_3 - \beta_1 - \beta_2 - \frac{1}{2}\beta_3$$

$$- \frac{1}{2} + \frac{3}{2}\beta_1 + \frac{5}{2}\beta_2 - 1$$

$$\alpha_1 = \beta_1 + \beta_2 - 2\beta_3$$

$$\Rightarrow \alpha_2 = (-1+3\beta_1+5\beta_2) + \frac{1}{2}(3+\beta_1-\beta_2-3\beta_3) - \frac{5}{2}$$

$$= -1 + 3\beta_1 + 5\beta_2 + \frac{3}{2} + \frac{1}{2}\beta_1 - \frac{1}{2}\beta_2 - \frac{3}{2}\beta_3 - \frac{5}{2}$$

$$= \frac{7}{2}\beta_1 + \frac{9}{2}\beta_2 - \frac{3}{2}\beta_3 - 2$$

$$\Rightarrow \alpha_3 = \frac{3}{2} + \frac{\beta_1}{2} - \frac{\beta_2}{2} - \frac{3}{2}\beta_3 - \frac{1}{2}$$

$$= \frac{\beta_1}{2} - \frac{1}{2}\beta_2 - \frac{3}{2}\beta_3 + 1$$

$$\Rightarrow \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -2 \\ 7/2 & 9/2 & -3/2 \\ 1/2 & -1/2 & -3/2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$