Cruz Rodriguez José Luis 10 27.10.04
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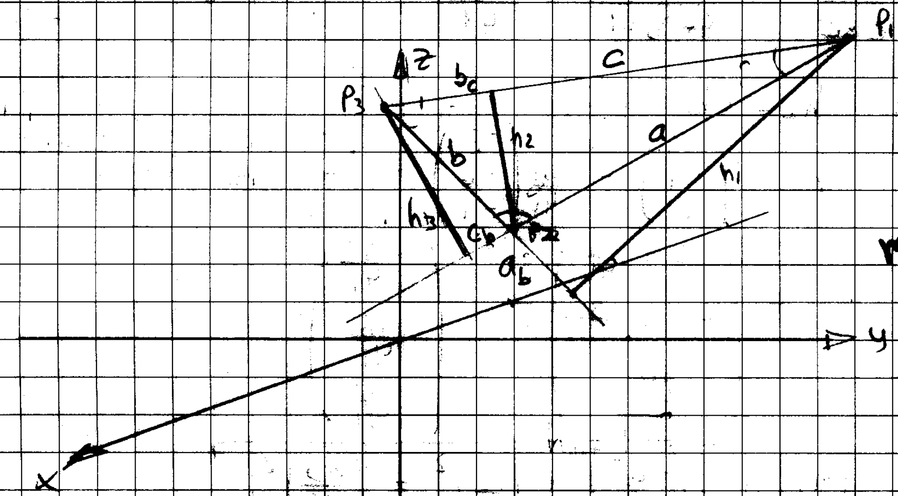
1.- Sean $P_1(-1,3,2)$, $P_2(0,1,1)$ y $P_3(-3,-3,1)$, calcular las alturas del triángulo $P_1P_2P_3$

Sean: $\overline{P_2P_1} = (-1-0, 3-1, 2-1) = (-1, 2, 1) = a$
$\overline{P_2P_3} = (-3-0, -3-1, 1-1) = (-3, -4, 0) = b$
$\overline{P_1P_3} = (-3-(-1), -3-3, 1-2) = (-2, -6, -1) = c$



muy bien!

Para h_1:

$$a_b = \alpha b$$
$$a_b \cdot a_b^{\perp} = 0 \quad -(1)$$

$$a_b^{\perp} = h_1 \Rightarrow h_1 = (a - a_b) = (a - \alpha b)$$

Sust. en (1)

$$\alpha b \cdot (a - \alpha b) = 0$$
$$ab\alpha - \alpha^2 bb = 0$$
$$ab - \alpha b \cdot b = 0$$
$$\alpha = \frac{ab}{b \cdot b} = \frac{3-8}{9+16} = \frac{-5}{25} = -\frac{1}{5}$$

$$\therefore \quad h_1 = a - \alpha b = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} - \alpha \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3\alpha \\ 4\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \\ 0 \end{pmatrix}$$

$$h_1 = \begin{pmatrix} -1 - \frac{3}{5} \\ 2 - \frac{4}{5} \\ 1 + 0 \end{pmatrix}$$

$$h_1 = \begin{pmatrix} -\frac{8}{5} \\ -\frac{14}{5} \\ 1 \end{pmatrix}, \quad |h_1| = \sqrt{(-\tfrac{8}{5})^2 + (-\tfrac{14}{5})^2 + 1^2} = 3.38$$

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Para h_2:

$$b_c = \beta c$$

$$b_c \cdot b_c^{\perp} = 0 \quad , \quad b_c^{\perp} = h_2$$

$$\beta = \frac{b \cdot c}{c \cdot c} = \frac{6+24}{4+36+1} = \frac{30}{41}$$

por otro lado: $h_2 = (b - \beta c)$

$$h_2 = \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} - \frac{30}{41}\begin{pmatrix} -2 \\ -6 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{60}{41} \\ \frac{180}{41} \\ \frac{30}{41} \end{pmatrix}$$

$$= \begin{pmatrix} -3 + \frac{60}{41} \\ -4 + \frac{180}{41} \\ \frac{30}{41} \end{pmatrix} = \begin{pmatrix} -\frac{63}{41} \\ \frac{16}{41} \\ \frac{30}{41} \end{pmatrix}$$

$$|h_2| = \sqrt{\left(-\frac{63}{41}\right)^2 + \left(\frac{16}{41}\right)^2 + \left(\frac{30}{41}\right)^2}$$

$$|h_2| = 1.75$$

Para h_3:

$$c_b = \gamma b$$

$$\gamma = \frac{c \cdot b}{b \cdot b} = \frac{30}{25}$$

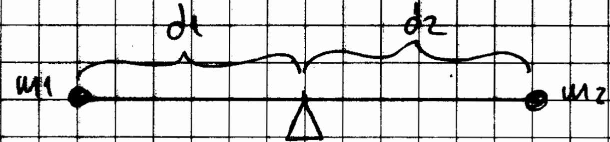
$$h_3 = (c - \gamma b) = \begin{pmatrix} -2 \\ -6 \\ -1 \end{pmatrix} - \frac{30}{25}\begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{90}{25} \\ \frac{120}{25} \\ 0 \end{pmatrix}$$

$$h_3 = \begin{pmatrix} -2 + \frac{90}{25} \\ -6 + \frac{120}{25} \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{40}{25} \\ -\frac{30}{25} \\ -1 \end{pmatrix}$$

$$|h_3| = \sqrt{\left(\frac{40}{25}\right)^2 + \left(-\frac{30}{25}\right)^2 + (-1)^2}$$

$$|h_3| = 2.24$$

2.- Consideremos el sistema



Dados d_1 y d_2, calcular m_1 y m_2 para que el sistema este en equilibrio.

Tenemos que:

$$m_1 d_1 = m_2 d_2 \quad \text{—} (1)$$

Entonces supongamos que $m_1 + m_2 = M \quad \text{—} (2)$

multiplicando por d_1 a (2)

$$m_1 d_1 + m_2 d_1 = M d_1$$
$$m_1 d_1 = M d_1 - m_2 d_1$$

Sustituyendo en (1)

$$M d_1 - m_2 d_1 = m_2 d_2$$
$$m_2 d_2 + m_2 d_1 = M d_1$$
$$m_2 (d_2 + d_1) = M d_1$$
$$\boxed{m_2 = \frac{M d_1}{d_2 + d_1}}$$

y de (1)

$$m_1 = m_2 \frac{d_2}{d_1}$$

$$m_1 = \frac{M \cancel{d_1}}{d_2 + d_1} \frac{d_2}{\cancel{d_1}}$$

$$\boxed{m_1 = \frac{M d_2}{d_2 + d_1}}$$