ROGELIO MONTERO CAMPOS

Sean los vectores $v_1 = \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix}$; $v_2 = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix}$ encontrar el vector w, talque $w \cdot v_1 = w \cdot v_2 = 0$

Tomamos $w \cdot v_2 = 0$ con $w = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \Rightarrow \alpha_1 + \alpha_2 - 7\alpha_3 = 0$

$\alpha_1 = 0 \Rightarrow \alpha_2 = 7, \alpha_3 = 1 \quad \begin{pmatrix} 0 \\ 7 \\ 1 \end{pmatrix}$; $\alpha_2 = 0 \Rightarrow \alpha_1 = 7, \alpha_3 = 1 \quad \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}$

Sabemos que un vector (w) es resultado de la combinación de 2 vectores no-coplanares. Entonces.

$$w = \beta_1 \begin{pmatrix} 0 \\ 7 \\ 1 \end{pmatrix} + \beta_2 \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}$$

Tomamos al vector v_1 $\quad v_1 \cdot w = 0 \quad \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix} \cdot \left[\beta_1 \begin{pmatrix} 0 \\ 7 \\ 1 \end{pmatrix} + \beta_2 \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}\right] = 0$

$\beta_1 \begin{pmatrix} 0 \\ 35 \\ -3 \end{pmatrix} + \beta_2 \begin{pmatrix} 28 \\ 0 \\ -3 \end{pmatrix} = 0 \qquad \beta_1(0+35-3) + \beta_2(28+0-3) = 0$

$\beta_1(32) + \beta_2(25) = 0 \qquad 32\beta_1 + 25\beta_2 = 0 \qquad \beta_1 = 25 \quad \beta_2 = -32$

$$w = 25 \begin{pmatrix} 0 \\ 7 \\ 1 \end{pmatrix} + (-32) \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 175 \\ 25 \end{pmatrix} + \begin{pmatrix} -224 \\ 0 \\ -32 \end{pmatrix} = \begin{pmatrix} -224 \\ 175 \\ -7 \end{pmatrix}$$

$$w = \begin{pmatrix} -224 \\ 175 \\ -7 \end{pmatrix}$$

w debe satisfacer:

a) $v_1 \cdot w = 0 \qquad v_1 = (4, 5, -3) \quad w = (-224, 175, -7)$

$4(-224) + 5(175) + (-3)(-7) = 0$

$-896 + 875 + 21 = 0$

$-896 + 896 = 0$ ✓

b) $v_2 \cdot w = 0 \qquad v_2 = (1, 1, -7) \quad w = (-224, 175, -7)$

$-224 + 175 + 49 = 0$

$-224 + 224 = 0$ ✓

$$w = \begin{pmatrix} -224 \\ 175 \\ -7 \end{pmatrix}$$

ROGELIO MONTERO CAMPOS.

Sean $V_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ $V_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ encontra un vector w tal que $v_1 \cdot w = 0$ y $v_2 \cdot w = 0$

Sea $w = (\alpha_1, \alpha_2, \alpha_3)$

$v_1 \cdot w = 0$

$x_1\alpha_1 + y_1\alpha_2 + z_1\alpha_3 = 0$ Si $\alpha_3 = 0$, $\alpha_1 = -y_1$, $\alpha_2 = x_1$ $\begin{pmatrix} -y_1 \\ x_1 \\ 0 \end{pmatrix}$

Si $\alpha_2 = 0$ $\alpha_1 = -z_1$ $\alpha_3 = x_1$ $\begin{pmatrix} -z_1 \\ 0 \\ x_1 \end{pmatrix}$

Pero $w = \beta_1 \begin{pmatrix} -y_1 \\ x_1 \\ 0 \end{pmatrix} + \beta_2 \begin{pmatrix} -z_1 \\ 0 \\ x_1 \end{pmatrix}$

$v_2 \cdot w = 0$ $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \cdot \left[\beta_1 \begin{pmatrix} -y_1 \\ x_1 \\ 0 \end{pmatrix} + \beta_2 \begin{pmatrix} -z_1 \\ 0 \\ x_1 \end{pmatrix}\right] = 0$

$\beta_1 \begin{pmatrix} -x_2 y_1 \\ x_1 y_2 \\ 0 \end{pmatrix} + \beta_2 \begin{pmatrix} -x_2 z_1 \\ 0 \\ x_1 z_2 \end{pmatrix} = 0$ $\beta_1(-x_2 y_1 + x_1 y_2) + \beta_2(-x_2 z_1 + x_1 z_2) = 0$

$\beta_1 = -x_2 z_1 + x_1 z_2$ $\beta_2 = x_2 y_1 - x_1 y_2$

$w = (-x_2 z_1 + x_1 z_2) \begin{pmatrix} -y_1 \\ x_1 \\ 0 \end{pmatrix} + (x_2 y_1 - x_1 y_2) \begin{pmatrix} -z_1 \\ 0 \\ x_1 \end{pmatrix} =$

$w = \begin{pmatrix} x_2 y_1 z_1 - x_1 y_1 z_2 \\ -x_1 x_2 z_1 + x_1^2 z_2 \\ 0 \end{pmatrix} + \begin{pmatrix} -x_2 y_1 z_1 + x_1 y_2 z_1 \\ 0 \\ x_1 x_2 y_1 - x_1^2 y_2 \end{pmatrix} =$

$$w = \begin{pmatrix} x_1 y_2 z_1 - x_1 y_1 z_2 \\ x_1^2 z_2 - x_1 x_2 z_1 \\ x_1 x_2 y_1 - x_1^2 y_2 \end{pmatrix} = \boxed{\begin{pmatrix} x_1 (y_2 z_1 - y_1 z_2) \\ x_1 (x_1 z_2 - x_2 z_1) \\ x_1 (x_2 y_1 - x_1 y_2) \end{pmatrix}}$$