

IV Encuentro Cuba- México de Métodos Numéricos y Optimización



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Análisis isogeométrico: un camino hacia la integración entre FEM y CAGD

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ICIMAF



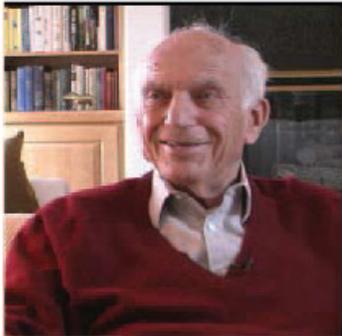


Temas de la charla

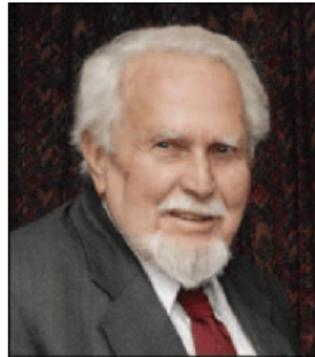
- Breve reseña histórica
- Análisis isogeométrico
- Espacio de funciones spline, base de B splines
- Formulación de un problema típico
- Ejemplo
- Comparación FEM vs IgA
- Proyectos, software
- Conclusiones

Reseña histórica sobre el análisis isogeométrico (IgA)

Método de elementos finitos (FEM)



R. Clough



O. C. Zienkiewicz

Diseño geométrico computarizado (CAGD)



P. Bezier



C. de Boor

- Comienza a desarrollarse en la década del 50 s. XX, aunque Courant publica un primer trabajo en 1943.
- Clough introduce en 1960 el término “elemento finito”.
- Zienkiewicz , 1968, utiliza “elementos isoparamétricos” (no proporcionan soluciones C_1 continuas).
- Década del 60 del siglo XX, P. Bezier y P. de Casteljaou utilizan los polinomios de Bernstein para diseñar curvas y superficies en la industria automotriz.
- Década del 70, s. XX - Teoría de las funciones B-splines , de Boor, Riesenfeld.
- Versprille 1975- Funciones NURBS, base de los sistemas de CAGD

Análisis isogeométrico (IgA)

- Es una disciplina nueva con apenas tiene un década de creada
- 2003 -Univ. de Texas, Tom Huges comienza a trabajar con sus estudiantes de doctorado A. Cottrell y Y. Bazilevs en la solución de algunos problemas diferenciales lineales utilizando funciones B spline racionales.



T. Hughes

2005 – Primer artículo

2009- Primer libro



Available online at www.sciencedirect.com



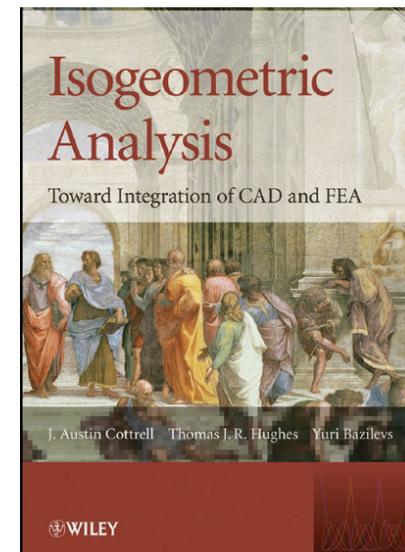
Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195

**Computer methods
in applied
mechanics and
engineering**

www.elsevier.com/locate/cma

Isogeometric analysis: CAD, finite elements, NURBS,
exact geometry and mesh refinement

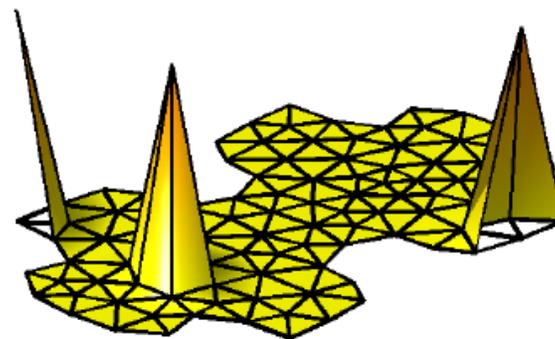
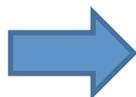
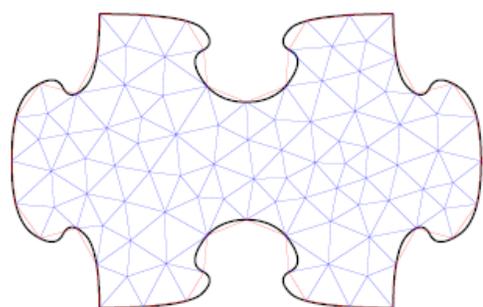
T.J.R. Hughes *, J.A. Cottrell, Y. Bazilevs



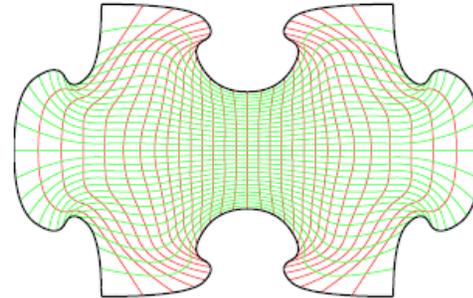
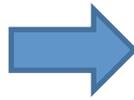
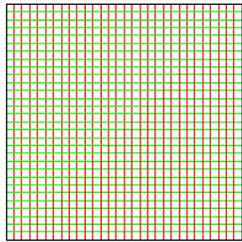
¿Cuál es el origen del problema?

FEM clásico

- Parte de una descripción de la frontera del dominio
- Discretiza el dominio lo cual introduce errores en la frontera
- Define las funciones básicas sobre el dominio discretizado



Análisis isogeométrico (IgA)



IgA

- Construye una parametrización del dominio mediante B-splines.
- La frontera se describe exactamente.
- Utiliza como funciones básicas el producto tensorial de B-splines definidos sobre el espacio de parámetros.

Esencia del IgA

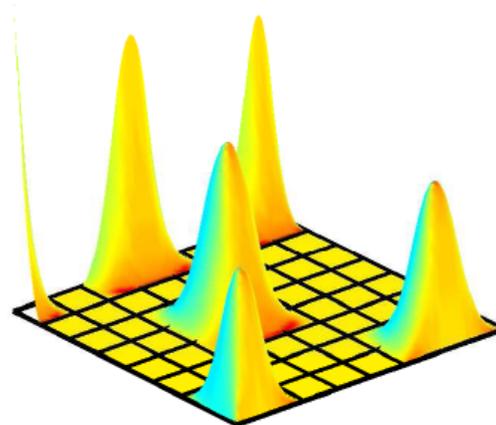
Funciones que se utilizan para describir la geometría del dominio

ISO



Funciones que se utilizan para aproximar la solución de la ecuación diferencial

NURBS
NonUniform Rational B-Splines



Funciones spline



$\mathbb{S}_{k,t}$ espacio de las funciones spline de grado k para la sucesión de nodos

$$t = \{t_1 \leq t_2 \leq \dots \leq t_n \leq t_{n+k+1}\} \quad \dim(\mathbb{S}_{k,t}) = n$$

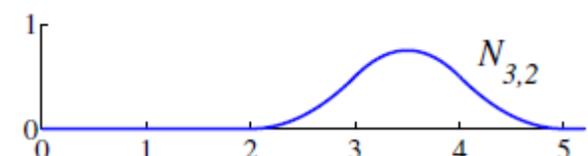
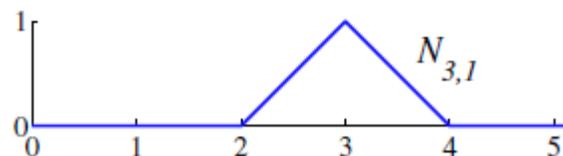
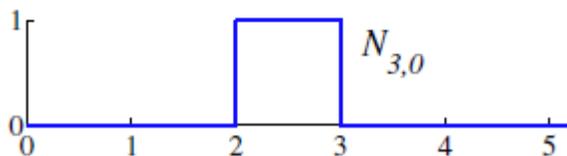
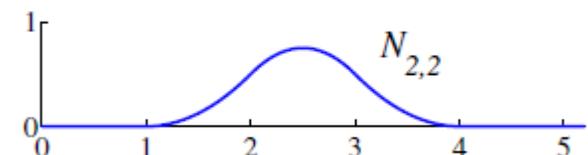
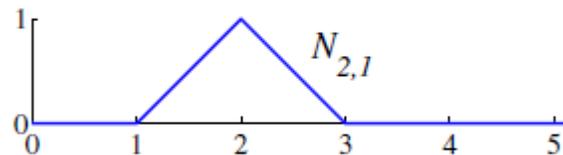
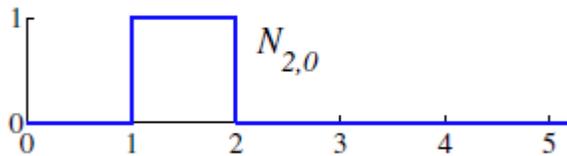
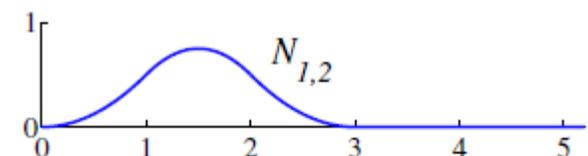
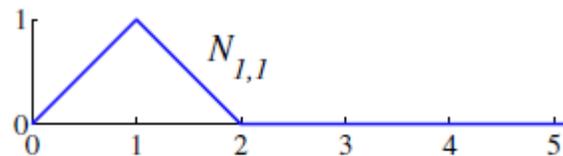
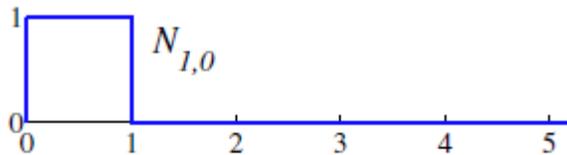
Funciones B-spline: base de $\mathbb{S}_{k,t}$

$$N_{i,0}(x) = \begin{cases} 1 & \text{si } t_i \leq x < t_{i+1} \\ 0 & \text{en otro caso} \end{cases}$$

Relación de recurrencia para los B-splines de grado k

$$N_{i,k}(x) = \frac{x - t_i}{t_{i+k} - t_i} N_{i,k-1}(x) + \frac{t_{i+k+1} - x}{t_{i+k+1} - t_{i+1}} N_{i+1,k-1}(x)$$

Algunos B-splines de grado 0, 1 y 2 para la sucesión de nodos uniforme $t = \{0, 1, 2, 3, 4\}$



Propiedades de los B-splines



- Entre dos nodos consecutivos son funciones polinomiales.

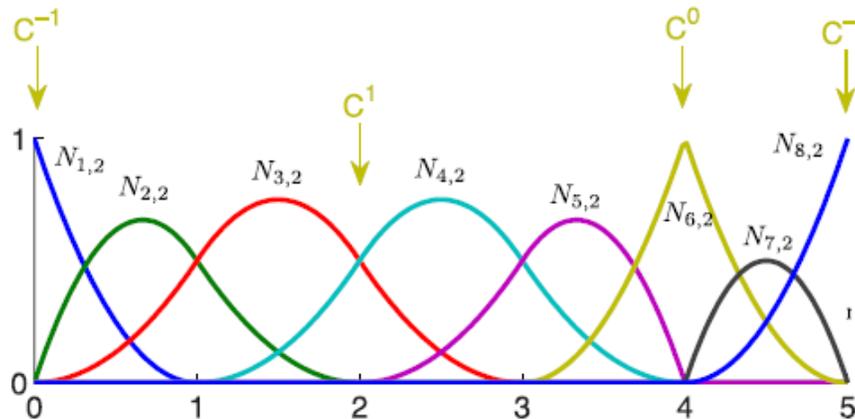
- $N_{i,k}(x) > 0$ para $t_i < x < t_{i+k+1}$

- $\sum_{i=1}^n N_{i,k}(x) = 1$ para toda $x \in [t_1, t_{n+k+1}]$

- Sólo hay $k+1$ B-splines de grado k que no se anulan entre dos nodos consecutivos. Para $x \in [t_j, t_{j+1}]$ estos son:

$$B_{j-k,k}, \dots, B_{j,k}$$

- Tienen $k-1$ derivadas continuas en los nodos simples.
- El número de derivadas continuas en un nodo de multiplicidad q es $k-q$



Funciones B-spline **cuadráticas**
para la sucesión de nodos

$$t = \{0, 0, 0, 1, 2, 3, 4, 4, 5, 5, 5\}$$

Curvas B-spline

$$C(x) = \sum_{i=1}^n \mathbf{P}_i N_{i,k}(x), \quad \mathbf{P}_i \in \mathbb{R}^2$$

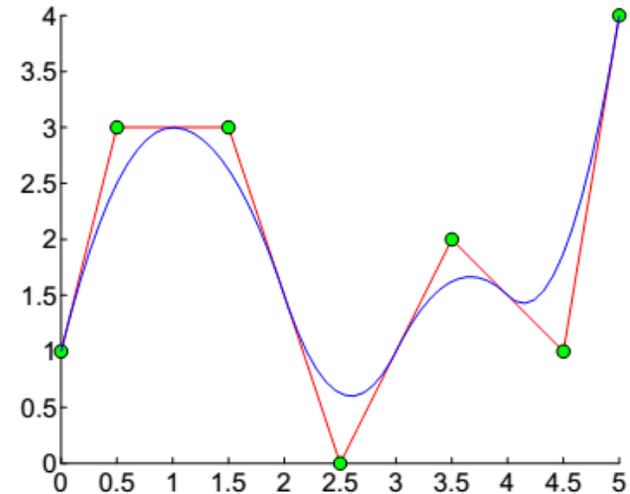
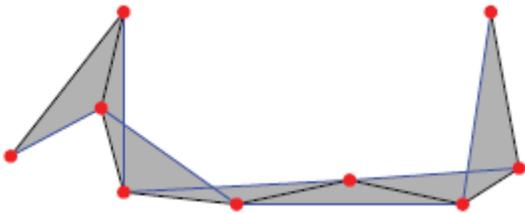


Curva B-spline cuadrática para la sucesión de nodos

$$t = \{0, 0, 0, 1, 2, 3, 4, 5, 5, 5\}$$

los puntos de control son los círculos verdes

k=2



Una curva spline se encuentra en el conjunto convexo definido como la unión de las envolturas convexas de k+1 puntos consecutivos del polígono de control.

Funciones NURBS

$$R_{i,k} = \frac{w_i N_{i,k}}{\sum_{j=1}^n w_j N_{j,k}}, \quad w_j \in \mathbb{R}$$

- Son las más utilizadas en CAD
- Permiten representar exactamente las cónicas
- Se definen a partir de las funciones B-splines

Mapeos B-spline producto tensorial



\mathcal{S}_{k,t^u} - Espacio de las funciones spline de grado k para la sucesión de nodos

$$t^u = \{t_1^u \leq t_2^u \leq \dots \leq t_n^u \leq t_{n+k+1}^u\}$$

$N_{1,k}(u), \dots, N_{n,k}(u)$ - base de \mathcal{S}_{k,t^u}

\mathcal{S}_{p,t^v} - Espacio de las funciones spline de grado p para la sucesión de nodos

$$t^v = \{t_1^v \leq t_2^v \leq \dots \leq t_m^v \leq t_{m+p}^v\}$$

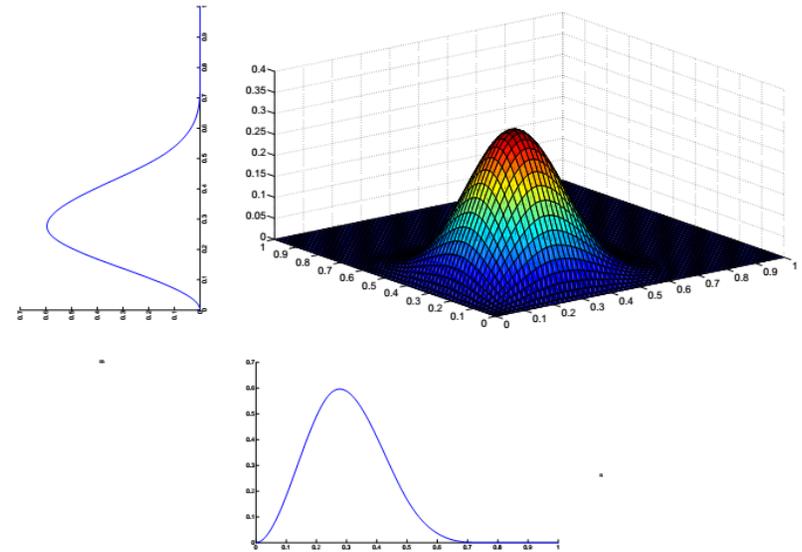
$M_{1,p}(v), \dots, M_{m,p}(v)$ - base de \mathcal{S}_{p,t^v}

B-spline producto tensorial definido en

$$[t_1^u, t_{n+k}^u] \times [t_1^v, t_{m+p}^v]$$

$$\mathbf{F}(u, v) = \sum_{i=1}^n \sum_{j=1}^m \mathbf{P}_{i,j} N_{i,k} M_{j,p} = \sum_{i=1}^n \sum_{j=1}^m \mathbf{P}_{i,j} B_{i,j}^{k,p}$$

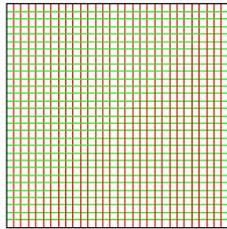
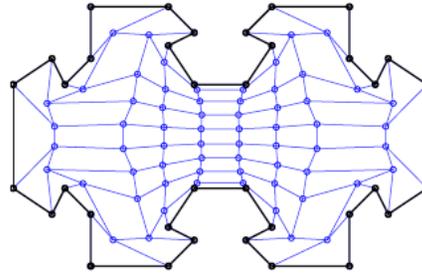
$\mathbf{P}_{i,j}$ - puntos de control, $B_{i,j}^{k,p} := N_{i,k} M_{j,p}$



B-spline bicúbico para la sucesión de nodos

$$t^u = t^v = \{0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1\}$$

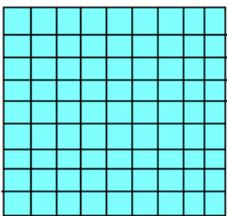
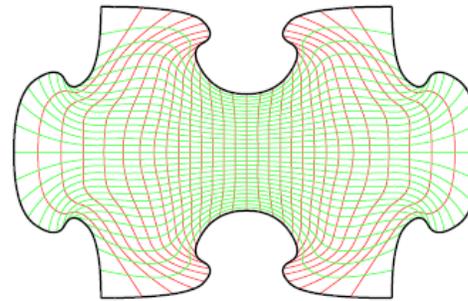
$$\mathbf{P}_{ij} \in \mathbb{R}^2,$$



$$\mathbf{F}(u, v)$$



mapeo plano

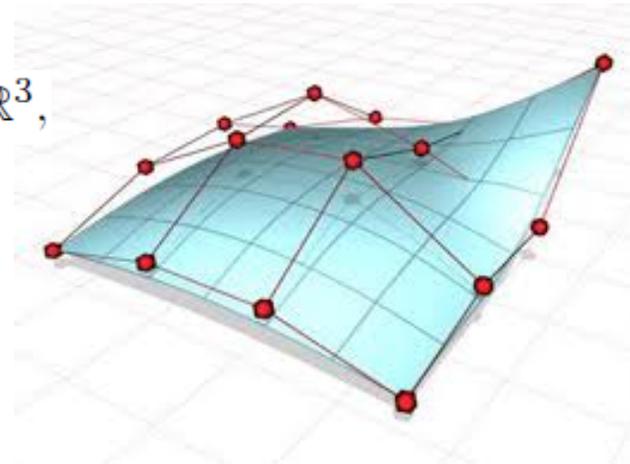


$$\mathbf{F}(u, v)$$



superficie paramétrica

$$\mathbf{P}_{ij} \in \mathbb{R}^3,$$



Solución de un problema clásico con IgA



Ecuación de Poisson

$$-\Delta g(x, y) = f(x, y), \quad (x, y) \in \Omega \subset \mathbb{R}^2$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{operador de Laplace}$$

condiciones de Dirichlet en la frontera $g(x, y) = 0, \quad \forall (x, y) \in \partial\Omega$

Formulación débil

$$\mathcal{H}^1(\Omega) = \{w \text{ tales que } w, \partial w / \partial x, \partial w / \partial y \in L^2(\Omega)\}$$

$$\mathcal{V} = \{w \in H^1(\Omega), \text{ tales que } w = 0 \text{ en } \partial\Omega\}$$

La solución $g \in \mathcal{V}$ es tal que $\forall w \in \mathcal{V}$

$$\int \int_{\Omega} -\Delta g w \, dx \, dy = \int \int_{\Omega} f w \, dx \, dy$$

utilizando el Teorema de Green el problema se transforma en:



Hallar $g \in \mathcal{V}$ tal que $\forall w \in \mathcal{V}$

$$\int \int_{\Omega} \nabla g \nabla w \, dx \, dy = \int \int_{\Omega} f w \, dx \, dy$$

Método de Galerkin con IgA



$\mathcal{V}_h \subset \mathcal{V}$ subespacio de dimensión finita
 $g, w \in \mathcal{V}_h$

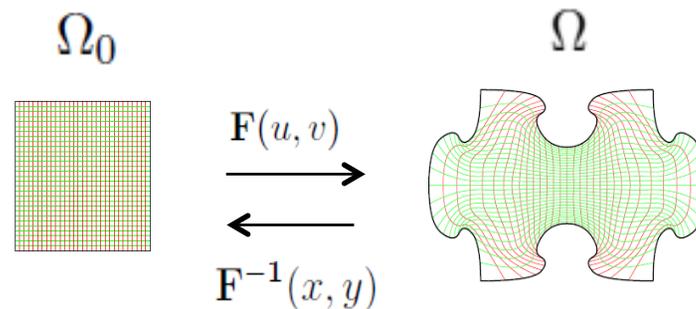
Enfoque isogeométrico

\mathcal{V}_h - espacio de funciones B-spline definidas sobre Ω que se anulan en la frontera

¿Cómo construir una base de \mathcal{V}_h

- Es preciso construir una parametrización de Ω

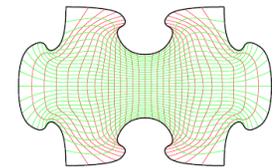
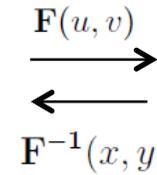
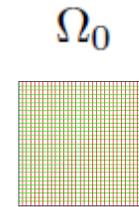
$$\mathbf{F}(u, v) = (x(u, v), y(u, v)) : \Omega_0 \rightarrow \Omega$$



Parametrización del dominio físico



$$\mathbf{F}(u, v) = \sum_{i=1}^n \sum_{j=1}^m \mathbf{P}_{i,j} B_{i,j}^{k,p} \quad \mathbf{P}_{i,j} \in \mathbb{R}^2 \text{ conocidos}$$



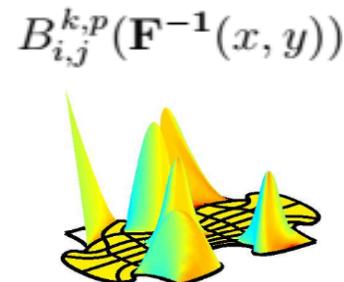
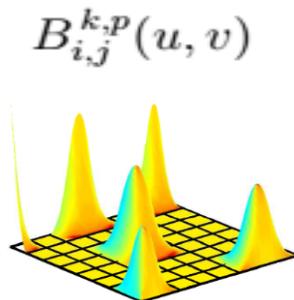
$$B_{i,j}^{k,p}(u, v) = N_{i,k}(u)M_{j,p}(v), \quad i = 1, \dots, n, \quad j = 1, \dots, m$$

$$N_{i,k}(u) \in \mathbb{S}_{t_u, k}, \quad i = 1, \dots, n$$

$$M_{j,p}(v) \in \mathbb{S}_{t_v, p}, \quad j = 1, \dots, m$$

Entonces

$$\mathcal{V}_h = \text{span} \left\{ \phi_l := B_{i,j}^{k,p}(\mathbf{F}^{-1}(x, y)), \quad l = 1, \dots, nm \mid \phi_l(\partial\Omega) = 0 \right\}$$



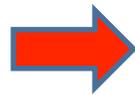
Método de Galerkin con IGA

$$\mathcal{V}_h = \text{span} \left\{ \phi_l := B_{i,j}^{k,p}(\mathbf{F}^{-1}(x, y)), l = 1, \dots, nm \mid \phi_l(\partial\Omega) = 0 \right\}$$

Hallar $g \in \mathcal{V}_h$ tal que:

$$\int \int_{\Omega} \nabla g \nabla \phi_l \, dx \, dy = \int \int_{\Omega} f \phi_l \, dx \, dy \quad \text{para } l = 1, \dots, nm$$

Pero
 $g \in \mathcal{V}_h$



$$g(x, y) = \sum_{i=1}^n \sum_{j=1}^m B_{i,j}^{k,p} (F^{-1}(x, y)) \mathbf{q}_{ij}$$

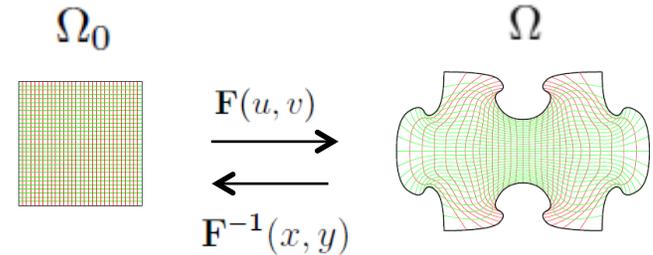
\mathbf{q}_{ij} **incógnitas**

Método de Galerkin con IgA, cont...



Teniendo en cuenta que:

$$\begin{aligned}\nabla_{(x,y)}g(x,y) &= J\mathbf{F}(u,v)^{-t} \nabla_{(u,v)}g(u,v) \\ \nabla_{(x,y)}\phi_l(x,y) &= J\mathbf{F}(u,v)^{-t} \nabla_{(u,v)}\phi_l(u,v)\end{aligned}$$



donde

$$J\mathbf{F}(u,v) = \begin{pmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{pmatrix}$$

el problema se reduce a trabajar en Ω_0 exigiendo que para $s = 1, \dots, nm$:

$$\int \int_{\Omega_0} (J\mathbf{F}(u,v)^{-t} \nabla g)(J\mathbf{F}(u,v)^{-t} \nabla \phi_s) |\det J\mathbf{F}(u,v)| du dv = \int \int_{\Omega_0} (f \phi_s)(\mathbf{F}(u,v)) |\det J\mathbf{F}(u,v)| du dv$$

Sustituyendo en esta ecuación $g(u,v)$ por su expresión en términos de la base de B-splines

$$g(u,v) = \sum_{l=1}^{nm} \mathbf{q}_l \phi_l(u,v)$$

Método de Galerkin con IgA, cont...



se obtiene el sistema de ecuaciones lineales

$$\mathbf{K}\mathbf{q} = \mathbf{f}$$

donde \mathbf{K} es la matriz de rigidez dada por:

$$\mathbf{K} = [K_{ls}]_{nm \times nm}, \quad K_{ls} = \int \int_{\Omega_0} J\mathbf{F}(u, v)^{-t} \nabla \phi_l (J\mathbf{F}(u, v)^{-t} \nabla \phi_s) |\det J\mathbf{F}(u, v)| du dv$$
$$l, s = 1, \dots, nm$$

$$\mathbf{f} = (f_1, \dots, f_{nm})^t, \quad f_s = \int \int_{\Omega_0} (f \phi_s)(\mathbf{F}(u, v)) |\det J\mathbf{F}(u, v)| du dv, \quad s = 1, \dots, nm$$

$$\mathbf{q} = (q_1, \dots, q_{nm})^t \quad \text{vector de las incógnitas}$$

Propiedades de la matriz de rigidez

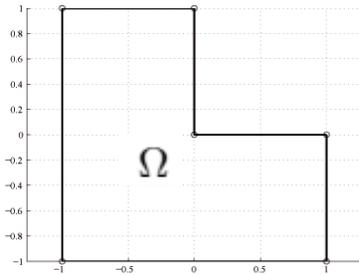
- **sparse** - pues los B-splines tienen soporte compacto
- **simétrica**

Ejemplo

$$-\Delta g(x, y) = f(x, y), \quad (x, y) \in \Omega \subset \mathbb{R}^2 \quad f(x, y) = 2\pi^2 \sin(\pi x) \sin(\pi y)$$

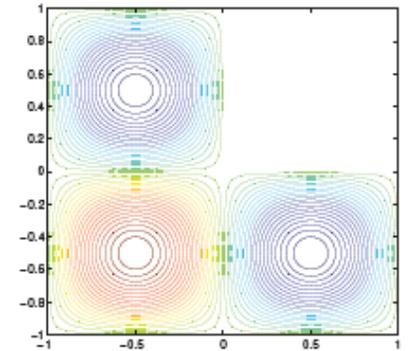
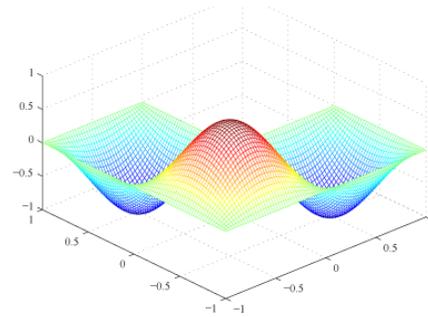
Condición de frontera

$$g(x, y) = 0, \quad \forall (x, y) \in \partial\Omega$$



Solución exacta

$$g(x, y) = \sin(\pi x) \sin(\pi y)$$



Solución aproximada utilizando IgA

- Construir una parametrización de Ω
 - Escoger el grado de las funciones básicas
 - Definir la sucesiones de nodos
 - Construir la malla de control del mapeo
- Calcular los puntos de control de la solución aproximada resolviendo el sistema de ecuaciones lineales

$$\mathbf{Kq} = \mathbf{f}$$

Solución aproximada desde un espacio de dimensión 12

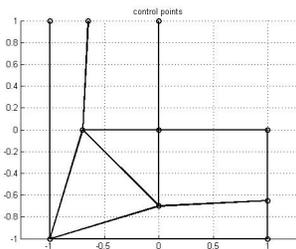
$$k = 2$$

$$t_u = \{0, 0, 0, 0.5, 1, 1, 1\} \quad \longrightarrow \quad n = \dim(\mathcal{S}_{t_u, k}) = 4$$

$$p = 2$$

$$t_v = \{0, 0, 0, 1, 1, 1\} \quad \longrightarrow \quad m = \dim(\mathcal{S}_{t_v, p}) = 3$$

12 puntos de control



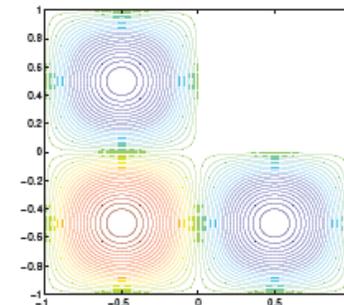
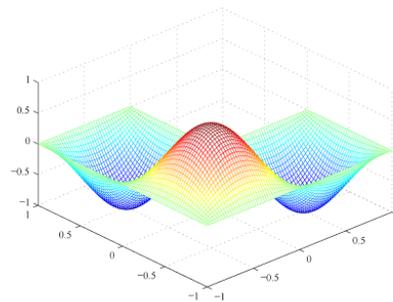
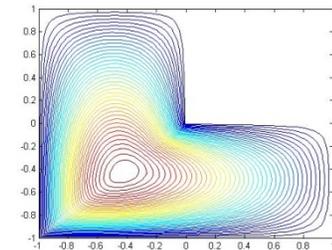
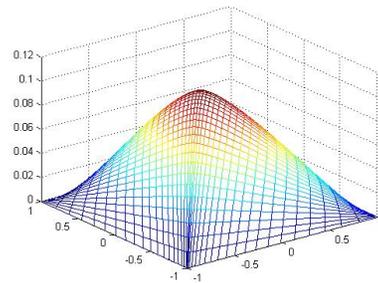
$$\mathbf{F}(u, v) = \sum_{i=1}^4 \sum_{j=1}^3 \mathbf{P}_{i,j} N_{i,2}(u) M_{j,2}(v)$$

Solución aproximada- Spline bicuadrático C1 continuo

$$g(x, y) = \sum_{i=1}^4 \sum_{j=1}^3 B_{i,j}^{k,p} (\mathbf{F}^{-1}(x, y)) \mathbf{q}_{ij}$$

Puntos de control $\mathbf{P}_{i,j}$

$$\begin{pmatrix} -1 & 1 \\ -1 & -1 \\ -1 & -1 \\ 1 & -1 \\ -0.65 & 1 \\ -0.7 & 0 \\ 0 & -0.7 \\ 1 & -0.65 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$



Solución aproximada desde un espacio de dimensión $k = 2$



$$t_u = \{0, 0, 0, 0.1250, 0.2500, 0.3750, 0.5, 0.6250, 0.7500, 0.8750, 1, 1, 1\}$$

$$p = 2$$

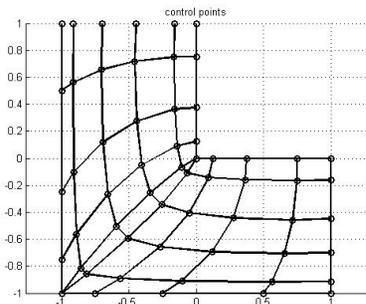
$$\longrightarrow n = \dim(\mathbb{S}_{t_u, k}) = 10$$

$$t_v = \{0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1\}$$

$$\longrightarrow m = \dim(\mathbb{S}_{t_v, p}) = 6$$



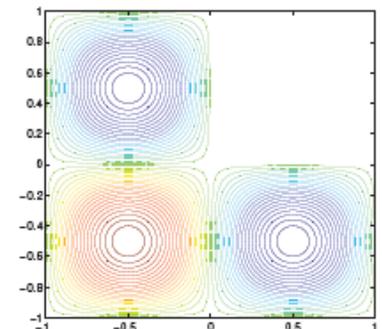
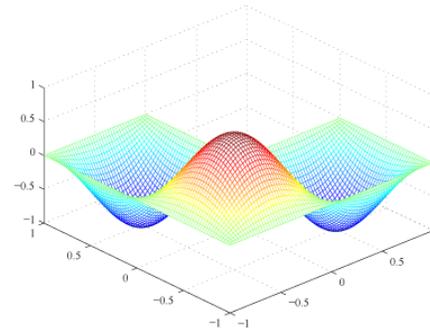
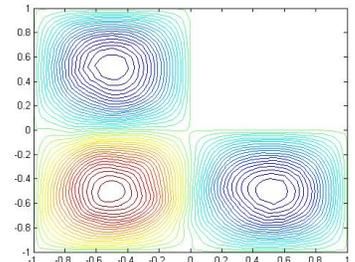
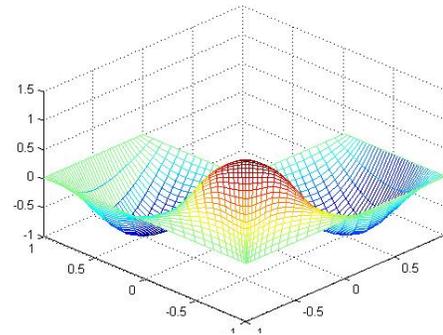
60 puntos de control



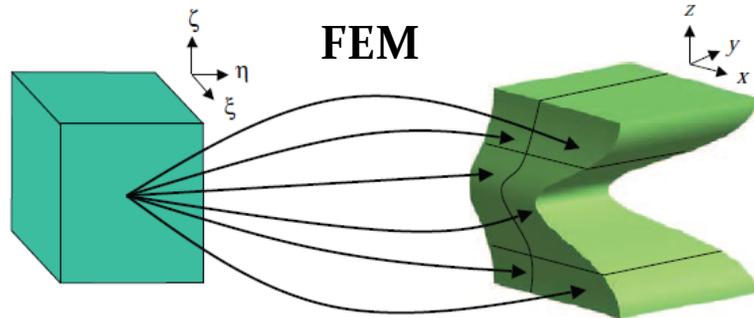
$$\mathbf{F}(u, v) = \sum_{i=1}^{10} \sum_{j=1}^6 \mathbf{P}_{i,j} N_{i,2}(u) M_{j,2}(v)$$

Solución aproximada- Spline bicuadrático C1 continuo

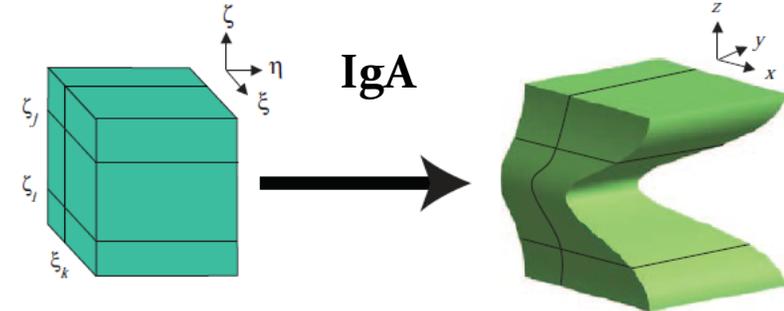
$$g(x, y) = \sum_{i=1}^{10} \sum_{j=1}^6 B_{i,j}^{k,p} (\mathbf{F}^{-1}(x, y)) \mathbf{q}_{ij}$$



Comparación FEM vs IgA



- No existe una función global de geometría y las funciones que se utilizan para representar la geometría son diferentes de las funciones que se utilizan para aproximar la solución del problema diferencial.
- La solución aproximada es una función polinomial a trozos de grado p , usualmente C^0 continua.
- Las incógnitas son los valores de la solución aproximada en ciertos puntos.



- La geometría se parametriza mediante una función **global** utilizando la misma base de funciones que se emplean para aproximar la solución del problema diferencial.
- La solución aproximada es una función polinomial a trozos de grado p y **suave**, C^{p-1} continua.
- Las incógnitas son los puntos de control que **no** son interpolados por la solución aproximada.

Tanto en FEM como en IgA la solución aproximada es una *combinación lineal* de las *funciones básicas* y por tanto la aplicación del método de Galerkin es esencialmente la misma.



Proyectos

The EXCITING project, EU project, 2012

EXaCt geometry simulaTIoN for optimized desiGn of vehicles and vessels

<http://exciting-project.eu/>

EC-Project EXCITING SCP8-2007-GA-218536

EXACT GEOMETRY SIMULATION
FOR OPTIMIZED DESIGN
OF VEHICLES AND VESSELS

<http://exciting-project.eu>



TERRIFIC- EU Project

Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies

2011-2014

<http://www.terrific-project.eu/>

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Isogeometric Discretizations in Continuum Mechanics

a FIRB "Futuro in ricerca" project

2010-2014

<http://www-dimat.unipv.it/sangalli/firb/index.html>



GeoPDE: Software en Octave compatible con Matlab



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GeoPDEs software

GeoPDEs is a suite of software tools for research on **Isogeometric Analysis** of PDEs. GeoPDEs is written in **Octave** and fully compatible with **Matlab**. It provides a common and flexible framework for implementing and testing new isogeometric methods in different application areas.

The suite consists of a set of interrelated **packages**. The main package, **geopdes_base**, defines the basic data-structures and methods, and should also serve as an entry point for understanding the implementation of an **Isogeometric Analysis** code.

Other packages deal with applications in **linear elasticity**, **fluid mechanics** and **electromagnetism**. A package specifically meant to allow handling **multipatch** NURBS geometries is also available.

Advances in Engineering Software 42 (2011) 1020–1034



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GeoPDEs: A research tool for Isogeometric Analysis of PDEs

C. de Falco^{a,*}, A. Reali^{b,c,d}, R. Vázquez^c



IsoGat: Software en Matlab

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ISOGAT

A tutorial 2D MATLAB code for solving elliptic diffusion-type problems, including Poisson's equation on single patch geometries, is presented. The basic steps of Isogeometric Analysis are explained and two examples are given. The code has a very lean structure and has been kept as simple as possible, such that the analogy but also the differences to traditional finite element analysis become apparent. It is not intended for large-scale problems.

URL: www-m2.ma.tum.de/bin/v...

Authors: Vuong, A.-V.; Heinrich, Ch.; Simeon, B.

Dependencies: Matlab

[Add information on this software.](#)

Keywords for this software



Similar software:

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Computer Aided Geometric Design 27 (2010) 644–655



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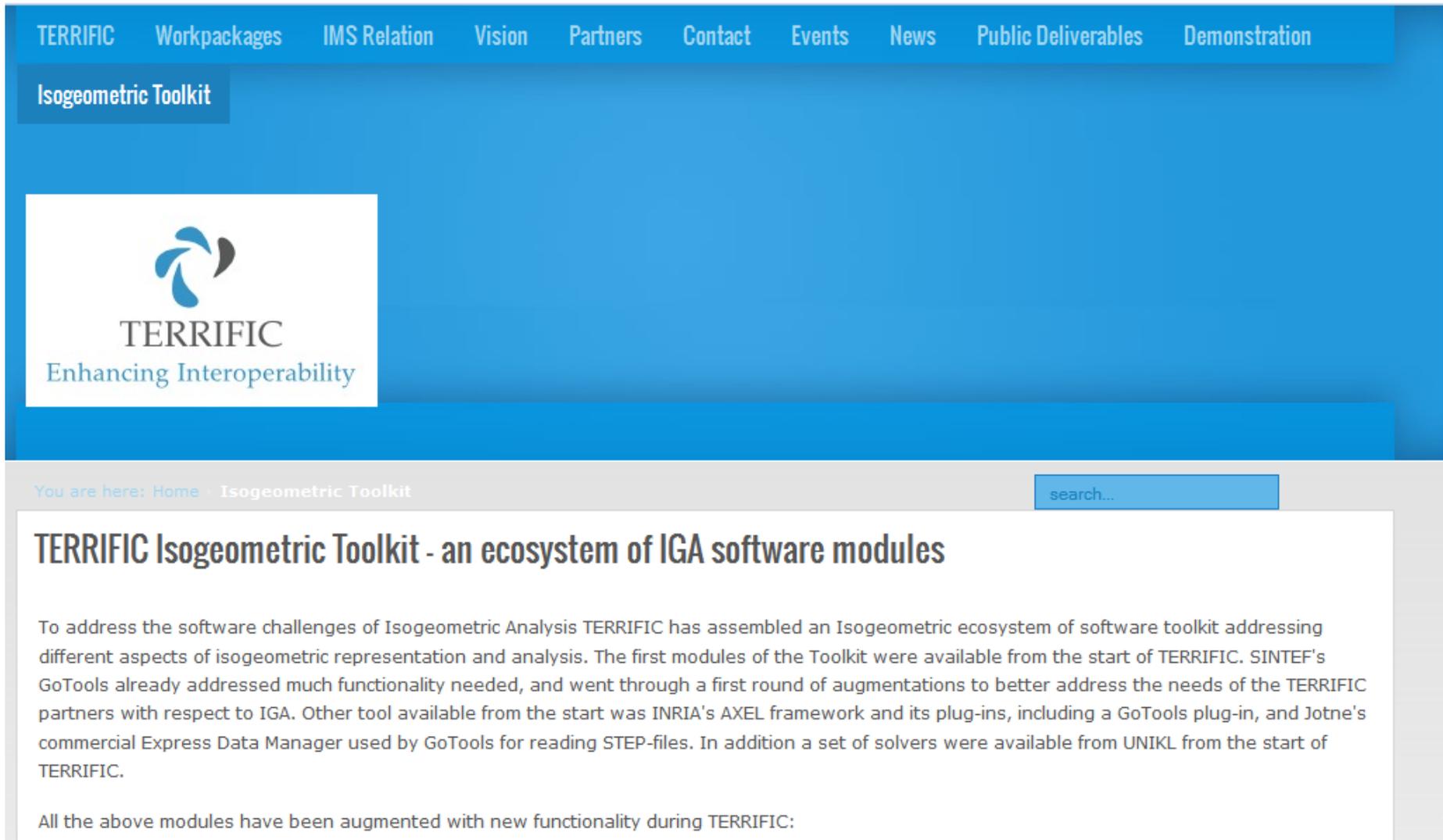
ISOGAT: A 2D tutorial MATLAB code for Isogeometric Analysis

A.-V. Vuong, Ch. Heinrich*, B. Simeon



TERRIFIC- EU Project

Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies



The screenshot shows the TERRIFIC website with a blue header containing navigation links: TERRIFIC, Workpackages, IMS Relation, Vision, Partners, Contact, Events, News, Public Deliverables, and Demonstration. A sub-header 'Isogeometric Toolkit' is highlighted. Below this is the TERRIFIC logo, which consists of a stylized blue and black circular icon above the text 'TERRIFIC' and 'Enhancing Interoperability'. A breadcrumb trail reads 'You are here: Home > Isogeometric Toolkit' and a search bar is visible. The main content area features the title 'TERRIFIC Isogeometric Toolkit - an ecosystem of IGA software modules' and a paragraph of text describing the toolkit's purpose and components.

TERRIFIC Workpackages IMS Relation Vision Partners Contact Events News Public Deliverables Demonstration

Isogeometric Toolkit

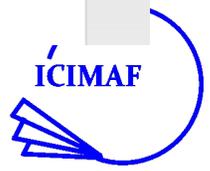

TERRIFIC
Enhancing Interoperability

You are here: [Home](#) > Isogeometric Toolkit

TERRIFIC Isogeometric Toolkit - an ecosystem of IGA software modules

To address the software challenges of Isogeometric Analysis TERRIFIC has assembled an Isogeometric ecosystem of software toolkit addressing different aspects of isogeometric representation and analysis. The first modules of the Toolkit were available from the start of TERRIFIC. SINTEF's GoTools already addressed much functionality needed, and went through a first round of augmentations to better address the needs of the TERRIFIC partners with respect to IGA. Other tool available from the start was INRIA's AXEL framework and its plug-ins, including a GoTools plug-in, and Jotne's commercial Express Data Manager used by GoTools for reading STEP-files. In addition a set of solvers were available from UNIKL from the start of TERRIFIC.

All the above modules have been augmented with new functionality during TERRIFIC:



abqNURBS: IgA con FEM-Abaqus, software en C++

abqNURBS

An implementation of NURBS based Isogeometric Analysis for Abaqus

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IsoGeometric Analysis



IsoGeometric Analysis is a recently developed computational approach that offers the possibility of integrating **finite element analysis (FEA)** into conventional **NURBS-based CAD** design tools. Currently, it is necessary to convert data between CAD and FEA packages to analyse new designs during development, a difficult task since the computational geometric approach for each is different. IsoGeometric analysis employs complex NURBS geometry (the basis of most CAD packages) in the FEA application directly. This allows models to be designed, tested and adjusted in one go, using a common data set. The pioneers of this technique are [Tom Hughes](#) and his group at the [University of Texas at Austin](#).

Despite its implementation in many research oriented codes, it currently exist very few software package freely available for usage and development (geodes developed at IMATI, Università di Pavia and Politecnico di Milano is

<http://abqnurbs.insa-lyon.fr/index.php/2-uncategorised/4-isogeometric-analysis>



Conclusiones

IgA

- es una herramienta muy poderosa para la solución numérica de PDE, que puede considerarse como una generalización del FEM clásico.
- Emplea las funciones básicas que se utilizan en CAD para representar la geometría Υ para construir la solución aproximada de PDE.
- es un área de investigación muy actual donde queda mucho por hacer:
 - Proponer algoritmos eficientes para parametrizar dominios con frontera irregular.
 - Obtener fórmulas de cuadratura especiales para las funciones spline.
 - Utilizar otras bases de funciones (B-splines generalizados, T-splines, etc)



Referencias



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A.V. Vuong, Ch. Heinrich, B. Simeon. ISOGAT: A 2D tutorial MATLAB code for isogeometric analysis. Computer Aided Geometric Design 27, 644-655, 2010.



C. de Falco, A. Reali, and R. Vázquez. GeoPDEs: a research tool for Isogeometric Analysis of PDEs. *Adv. Eng. Softw.*, 42(12):1020-1034, 2011.



VP Nguyen, S Bordas, T Rabczuk. [Isogeometric analysis: An overview and computer implementation aspects](#), Elsevier Science, 2013.

