Lógica Computacional. Tarea 3

1. Realiza las siguientes sustituciones:

   (a) $(((\forall x\,.\,(\exists y\,.\,P_2^2(x,z)) \vee P_2^1(y)))_{[x:=f_1^1(z)]})_{[y:=f_3^2(z,x)]}$;

   (b) $(((\exists x\,.\,(\forall z\,.\,(\exists y\,.\,P_1^3(x,y,z)))) \Leftrightarrow P_2^3(x_1,y,z_2))_{[x_1:=f_1^3(x,y,z_3)]})_{[z:=f_2^1(y)]}$;

   (c) $((\exists x\,.\,P_1^3(x,y,z)) \Rightarrow P_1^1(x))_{[z:=f_1^2(x,y)]}$;

   (d) $(((\forall y\,.\,P_1^3(x,y,z)) \wedge (\exists z\,.\,P_2^3(x,y,z)) \wedge (\forall x\,.\,P_3^3(x,y,z)))_{[x:=f_1^2(y,z)]})_{[y:=f_2^2(x,z)]}$.

2. Encuentra un modelo para el siguiente conjunto de fórmulas (justifica tu respuesta):
   $\forall x\,.\,(\exists y\,.\,P_1^2(y,x))$, $\forall x\,.\,\neg P_2^2(x,c) \Rightarrow P_1^2(x,c)$, $\neg\exists x\,.\,P_2^2(c,f_1^1(x))$, $\neg\exists x\,.\,P_2^2(x,f_1^1(x))$.

3. Demuestra los siguientes teoremas de deducción natural:

   (a) $\forall x\,.\,(\exists y\,.\,P_1^1(x) \Rightarrow P_2^1(y)) \vdash_N \neg(\exists x\,.\,(\forall y\,.\,P_1^1(x) \wedge \neg P_2^1(y)))$;

   (b) $\forall x\,.\,\forall y\,.\,P_1^1(x) \Rightarrow P_2^1(x) \wedge P_3^1(y), P_1^1(c) \vdash_N \neg\forall z\,.\,\neg P_2^1(z)$;

   (c)$\neg(\exists x\,.\,(P_1^1(x) \wedge (\forall y\,.\,\neg P_1^2(x,y)))) \vdash_N \forall x\,.\,(P_1^1(x) \Rightarrow (\exists y\,.\,P_1^2(x,y)))$;

   (d) $\forall x\,.\,P_1^1(x) \Rightarrow (\forall y\,.\,P_2^1(y) \Rightarrow (\forall z\,.\,P_3^1(z) \Rightarrow (P_4^1(z) \Rightarrow (\exists x_1\,.\,P_1^2(z,x_1)))))$, $P_1^1(c)$, $P_2^1(c)$, $P_3^1(c) \vdash_N (\forall x_1\,.\,\neg P_1^2(c,x_1)) \Rightarrow \neg P_4^1(c)$.

4. Demuestra que la regla de deducción natural $I\forall$ es correcta.