



Craig Kaplan

Nota. Estimados lectores, el 20 de marzo del año pasado sucedió algo extraordinario. Un grupo de cuatro matemáticos dio a conocer la solución a un famoso problema de teselaciones.

Una teselación es un patrón de figuras que cubren completamente una superficie plana. Esta cubierta satisface dos condiciones: No deja espacios sin cubrir.

Las figuras utilizadas no se superponen. Llamemos \mathbb{R}^2 al plano euclidiano.

Esta superficie plana tiene una infinidad de teselaciones. Si restringimos la familia de teselaciones de \mathbb{R}^2 a aquellas en las que se utiliza una sola figura, de todos modos hay muchísimas teselaciones posibles.

Por ejemplo, utilizando solamente cuadrados; o utilizando solamente triángulos equiláteros, o utilizando solamente hexágonos regulares. En el número 798 de este Boletín se mencionó una teselación de \mathbb{R}^2 utilizando un sólo pentágono no regular convexo.

Todas las teselaciones mencionadas tienen la propiedad de ser periódicas. Es decir, crean un dibujo que, bajo traslaciones de \mathbb{R}^2 , se mantiene fijo. Al aplicar una traslación conveniente en el plano, obtienes la misma figura.

Bien, resulta que David Smith, Craig Kaplan, Joseph Samuel Myers, y Chaim Goodman-Strauss, encontraron una figura (que llamaron the hat) con la cual se obtiene una teselación del plano que no es periódica. Es decir, que dada cualquier traslación de \mathbb{R}^2 la figura resultante es distinta a la figura original.

El mundo, al menos el mundo de los que les gustan las teselaciones, quedó asombrado. Este sombrero responde a un problema abierto que llevaba décadas sin solución.

Reproducimos a continuación una entrevista a Craig Kaplan realizada por Tim Chartier. En ella nos enteramos de la increíble ruta que llevó a este descubrimiento.

El texto apareció el 4 de mayo de 2023 en el blog Math Values, contenido en la página de la Mathematical Association of America.

<https://www.mathvalues.org/>

Unlocking the aperiodic monotile's secrets An interview with Craig Kaplan

By Tim Chartier

On Monday, March 20, 2023, a paper *An aperiodic monotile* appeared on the arxiv (<https://arxiv.org/abs/2303.10798>). It introduced a 13-sided polygon called *the hat* that tiled the plane aperiodically. Simply put, the research result is huge. The news not only spread quickly among mathematicians, but it also spread to the public at large. Soon, an article appeared in *The New York Times*. To learn more about this result and the journey to its discovery, *Math Values* interviewed one of the researchers, Craig Kaplan.

Tim Chartier: First, can you explain what an aperiodic tile is and why it is called an einstein tile?

Craig Kaplan: The name “einstein” is a pun attributed to Ludwig Danzer, literally meaning “one stone” in German but loosely translated as “one shape” or “one tile.” More formally, an einstein is an “aperiodic monotile”; let’s take that phrase one word at a time.

Every parallelogram tiles the plane in a very simple way, by arranging copies of the parallelogram into infinite rows and columns, forming a kind of skewed grid. A “periodic” tiling is one that repeats with the same pattern: it consists of copies of some finite patch of tiles, stamped out endlessly in regular intervals given by two translation vectors.

Now, given a set of shapes, they may or may not admit periodic tilings, and they may or may not admit non-periodic tilings. A set of shapes is called “aperiodic” when they admit tilings, but none that are periodic in the sense described above. That is, the shapes are well-behaved enough to allow infinite tilings, but always manage to disrupt the regularity of translational symmetry along the way. Note that aperiodicity is a property of “a set of shapes”, and not of any particular tiling they might admit.

“Monotile”, like “einstein”, just means “one shape”. We’ve known of aperiodic sets of shapes for decades, but our work was the first to exhibit a set of size one, hence “aperiodic monotile.”

Tim Chartier: Can you give us a broad overview of the history of the search for aperiodic tilings?

Craig Kaplan: It was only in the 1960s that Hao Wang conceived of the idea of an aperiodic set of shapes, and immediately declared them to be impossible (a completely reasonable supposition at the time!). A few years later, Robert Berger exhibited the first aperiodic set, one containing over 20,000 distinct shapes. Naturally, mathematicians sought ever smaller aperiodic sets from that point onward. Optimizations of Berger’s work whittled the minimum size down to around 100 shapes, and then to Raphael Robinson’s eminently manageable set of size six in 1971.

Penrose’s famous “kite and dart” tiles revolutionized the field with a set of size two. Since then, other small aperiodic sets of various sizes were discovered, including sets of size two by Robert Ammann, my co-author Chaim Goodman-Strauss, and others. Some monotiles have even been proposed, including a wonderful aperiodic hexagon by Joan Taylor and Joshua Socolar, but these required extra constraints that couldn’t be expressed by shape alone. Overall, each of these sets has been a fresh, one-off discovery. We have very few general principles or techniques that tell us how or where to look for aperiodicity. For about 50 years, then, the search for small aperiodic sets was stalled at two, and mathematicians have wondered about the possibility of an aperiodic monotile.

Tim Chartier: How was the tile found? How did you possibly sense it might be aperiodic or was that even the initial inkling?

Craig Kaplan: David Smith enjoys experimenting with shapes as a hobby. He'll pick a simple shape, say a polyform made by gluing copies of some unit cell together, and investigate visually appealing ways to make that shape tile the plane. Often he'll use a computer-controlled craft cutter to cut copies of the shape out of paper and manipulate them by hand.

In practice, most shapes either tile in a simple way (e.g., it only takes one or two of them to create a patch that tiles in a grid by translation), or fail to tile in a simple way (e.g., there's a spot on the shape's boundary where no neighboring copy of the shape can be placed). David noticed immediately that the "hat" didn't follow this pattern: he was able to assemble substantial patches of hats, without ever discerning a clear periodic pattern in them. He's experienced enough playing this game that he knew "something" interesting was happening, which is why he started reaching out to others.

What caught my eye immediately was the sparse, even spacing of reflected tiles, in a variety of orientations, in the patches David had constructed by hand. It seemed clear that there were hidden rules at play, forcing him to inject occasional reflected tiles so that he could keep building outward. That's something I hadn't seen before, and it was highly suspicious. I started generating larger patches computationally, and they exhibited the same pattern, which was compelling evidence that we were seeing something truly new.

Tim Chartier: So, David Smith found an oddly behaving tile and connected with you. In time, you brought in Joshua Samuel Myers and Chaim Goodman-Strauss. Can you walk us through moments in that growing collaboration?

Craig Kaplan: David first reached out to me on November 17th to ask about some earlier work I had done computing what are called "Heesch numbers" (a kind of measure of the complexity of shapes that don't tile the plane). He wanted to know whether the software I had created could be used to compute large patches of hats automatically. My initial reaction was to stall for time - I was close to the end of term and wanted to finish teaching before I turned my attention to his shape! Fortunately he persisted, sending me pictures of paper patches he had constructed, and pretty soon I had caught "hat-fever". On November 24th he dared to suggest that the shape could be an einstein, and from that point onward I spent just about every spare moment exploring the "hat" through computation and digital drawings. I begged David to let me keep working on the "hat" solo through the holidays, just to savor it a little longer, and that we'd reach out to others in the new year.

We contacted Chaim Goodman-Strauss in early January and Joseph Myers in mid-January. By that time I had arrived at a construction that could tile the plane with hats,

but we were definitely going to need more mathematical muscle to complete a full proof of aperiodicity. Happily, we didn't have that long to wait. Eight days after we reached out to him, on January 25th, Joseph came back with a full proof of aperiodicity based on my construction! It still amazes me that all the pieces of this work fell into place so quickly -that's certainly not something I've ever experienced before, nor anything we should expect in mathematical research.

Tim Chartier: Before your result, we didn't know if there was one einstein tile. Your paper shows a continuum. Can you outline the steps in that discovery?

Craig Kaplan: Back in December, David emailed me to mention that he was experimenting with a second shape that was also behaving strangely. This other shape was a turtle-shaped union of ten kite-shaped cells (rather than the eight making up the hat). My reaction was to put this shape on the back burner; it looked interesting, but it seemed natural to me to let our study of the "hat" run its course before turning to it. I did compute some large patches of turtles, but mostly I tried not to let it haunt me as I played with the "hat".

In January, David shared the turtle with Chaim and Joseph, who happily weren't as dismissive as I was! What followed was a sequence of stunning revelations by Joseph. First, the "turtle" was also aperiodic, because it tiled in a manner that was equivalent to the "hat". Second, the "hat" and "turtle" were just two points belonging to a continuum of equivalent aperiodic monotiles. And third (if that weren't already enough), the continuum itself could be harnessed to produce a completely new style of proof of aperiodicity! These ideas really elevated the work to a higher plateau of sophistication.

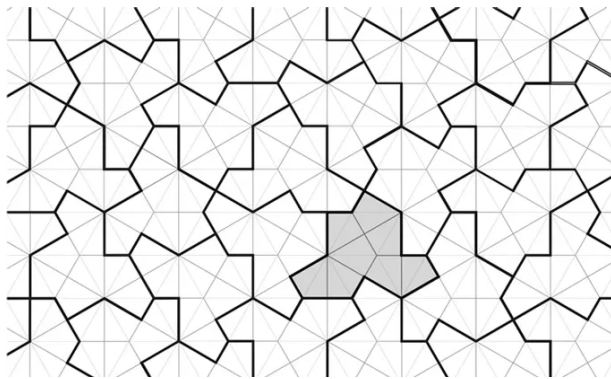
Tim Chartier: Your paper was released on the ArXiv and quickly got attention far beyond people in your field. Can you share what the media coverage was like?

Craig Kaplan: The response has been exhilarating, heartwarming, and a little overwhelming.

I suppose it's newsworthy any time a longstanding open problem is solved (for example, the week before we published our paper, there were many headlines about a major advance in Ramsey theory). But on top of that, I think our work probably has an extra dash of public appeal. There's an immediacy to the result: anybody can examine a drawing of the tiling and see its unusual behavior, unimpeded by layers of abstraction or notation. Also, there's the alluring narrative of the initial discovery by a hobbyist. I hope David will inspire others to experiment with mathematical ideas just for the fun of it.

I've also enjoyed talking to and working with science communicators and other journalists from around the world. Generally they've been enthusiastic about the topic, eager to understand the details of our paper, and determined to represent the work faithfully to the public. I'm grateful for all of that.





The hat, el sombrero.

Tim Chartier: People have created artwork. Are there pieces you can share? What's it like to see your work so quickly inspire so many worldwide?

Craig Kaplan: Despite the purely mathematical focus of this work, most of my academic research is more interdisciplinary, and concerns applications of mathematics and computation to art and design. So I've been absolutely thrilled to see so many people inspired to make their own art based on the "hat". Many people are fabricating their own hats (a friend at another university complained that he couldn't get access to the 3D printers because they were booked solid with students printing hats). There have been numerous threats of renovated bathroom floors, and I'm looking forward to the first person who makes good on that. 🍷

Carrera Atlética 85 aniversario de nuestra Facultad

Lugar y fecha: Se realizará, el próximo **domingo 8 de septiembre a las 8:30 horas**, en Ciudad Universitaria.

Ramas, categorías y distancias: Femenil y varonil, juvenil, libre (18 a 39 años), máster (40 a 49 años) y veteranos (50 años en adelante), 8.5 kilómetros.

Inscripciones: A partir de la publicación de la presente hasta el **30 de agosto**, en Tienda en línea de la UNAM.

<https://www.tiendaenlinea.unam.mx/>

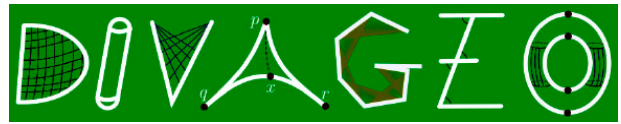
Costo: Del 18 de agosto y hasta el 30 de agosto, para la Comunidad de la Facultad de Ciencias \$350.00, otros universitarios y externos \$400.00.

Recorrido: Salida y meta: Facultad de Ciencias, Facultad de Veterinaria, Anexo de Ingeniería, Facultad de Contaduría, Estadio *Tapatío Méndez*, Campo de Béisbol, Pista de calentamiento, Jardín Botánico y Facultad de Ciencias.

Más información:

<https://www.fciencias.unam.mx/>

Seminario



*El camino hacia la ϵ -gravedad
cuántica canónica*

Omar Corona Tejada
Facultad de Ciencias, UNAM

Resumen. *En la búsqueda de una teoría de unificación de la Relatividad General y la Mecánica Cuántica se han presentado diferentes enfoques. Estas teorías deben cumplir algunos requisitos deseables:*

- 1) Preservar la Teoría Cuántica de Campos, es decir, la teoría cuántica vigente para la relatividad del espaciotiempo de Minkowski.*
- 2) Preservar en cierta medida a la Mecánica Cuántica y a la Relatividad General ya que son las teorías vigentes y bien fundamentadas.*
- 3) Describir la dinámica de forma cuanto-relativista y concordar con la parte experimental.*

En esta plática se revisarán los aspectos de la teoría de ADM, cómo surgen y una ligera generalización.

Viernes 23 de agosto, 11:00 am.

Información de Zoom: ID reunión: 850 7703 4297
Clave de acceso: 660866

O en el enlace

<https://cuaieed-unam.zoom.us/j/85077034297?pwd=N3A0ZHc1VE1pOGpXMUJtcWEwNmVPQT09>

Organizan

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Wikipedia dice: *El formalismo ADM, nombrado por sus autores Richard Arnowitt, Stanley Deser y Charles W. Misner, es una formulación hamiltoniana de la relatividad general que juega un papel importante en la gravedad cuántica y en la relatividad numérica. Fue publicado por primera vez en 1959.*

