The Double Life of Felix Hausdorff / Paul Mongré
Segunda parte

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Nota. *Estimados lectores, reproducimos a continuación la segunda parte del artículo*

The Double Life of Felix Hausdorff, Paul Mongré

Este trabajo fue escrito por Walter Purkert, y apareció publicado en la revista The Mathematical Intelligencer *en abril de 2009.*
El texto nos ofrece un vistazo rápido a una biografía realmente extraordinaria.
Estudia a un habitante del siglo XX que decidió llevar una doble vida. Bajo el nombre de Paul Mongré escribió un libro de poesía, un libro de aforismos, una obra de teatro y diversos ensayos literarios.
Como Felix Hausdorff realizó una obra matemática enorme y sorprendente.
Es cierto que en el descubrimiento y la creación de la colección gigantesca de resultados que englobamos en la palabra "matemáticas" han participado miles de personas. Darle crédito a todas ellas es una tarea imposible. Sin embargo, si nos restringimos al siglo XX, de inmediato salta ante nuestra vista la participación de Felix Hausdorff. No es exagerado decir que su influencia ha llegado a la mayoría de todos nosotros, los que nos dedicamos a la enseñanza, la investigación y el aprendizaje de la topología, la teoría de conjuntos y el análisis matemático.
En verdad, Felix Hausdorff fue uno de los miembros de nuestra comunidad cuya aportación despierta una gran admiración, junto con un profundo agradecimiento.
La primera parte de este trabajo se encuentra en el número 788 del Boletín.
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La versión completa del artículo se puede consultar en la página:

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In 1899 Hausdorff married Charlotte Goldschmidt, the daughter of a Jewish physician, Siegismund Goldschmidt, from Bad Reichenhall. This man's stepmother, incidentally, was the famous feminist and preschool pedagogue, Henriette Goldschmidt. In 1900 the Hausdorffs' daughter Lenore (Nora), their only child, was born; she survived through the Nazi era and died in 1991 in Bonn. In December of 1901 Hausdorff was appointed as an unofficial associate professor at Leipzig University. In submitting the faculty's proposal for this appointment, which contained a very favorable assessment given by his colleagues and composed by Heinrich Bruns, the Dean added the following remark:

The faculty considers itself, however, duty bound to inform the Royal Ministry that the present proposal was not approved by all members in the meeting on the 2nd of November this year, but rather by a vote of 22 to 7. The minority who voted against Dr. Hausdorff did so because he is of the Jewish faith.

This amendatory remark illuminates at a glance the open anti-semitism that was especially on the rise across the entire German Empire after the financial crash that followed its founding in 1871. Leipzig was at the center of the anti-Semitic movement, in which students played a large role. This may well have been one reason why Hausdorff never felt particularly comfortable teaching there; another reason was the strong sense of hierarchy among full professors who tended to disregard their junior colleagues.

Hausdorff mathematical research covered unusually broad terrain: he wrote papers on such diverse topics as optics (1896), non-Euclidean geometry (1899), hyper-complex number systems (1900), insurance mathematics (1897), and probability theory (1901). Hausdorff's principal field of research, however, soon became set theory, especially the theory of ordered sets. Initially it was his philosophical interests that led him to Cantor's ideas.

In the summer semester of 1901, Hausdorff offered a lecture course on set theory; this was nearly the first in Germany, only Ernst Zermelo's course in Gottingen the previous semester preceded it. (Cantor himself never offered lectures on set theory in Halle).

In the summer semester of 1910, Hausdorff was appointed to a position as official associate professor at Bonn University. There he had not taught any course in set theory since 1901, even though this was his primary field of research. After arriving in Bonn, he immediately gave a course on set theory, which he repeated in the summer of 1912 in a revised and expanded form. It was during that summer that Hausdorff began work on his magnum opus, *Grundzüge der Mengenlehre*. He completed it in Greifswald, where he began teaching as a full professor in the summer semester of 1913; his book appeared in print in April 1914.

Set theory, as this area of mathematics was understood at the time, included not just the general theory of sets but also point sets and the theories of content and measure. Hausdorff's work was the first textbook that dealt systematically with all aspects of set theory in this comprehensive sense and which provided complete proofs in a masterful form. Moreover, it went well beyond the presentation of known results: it contained a number of significant original contributions by its author.

The first six chapters of the *Grundzuge* deal with general set theory. Hausdorff begins by setting out an algebra for sets that includes some new concepts that would prove influential. These introductory paragraphs on sets and their operations also contain the modern set-theoretic concept of a function; here we encounter, so to speak, many of the ingredients that form the modern language of mathematics.

The chapters on "point sets" -one might better say topology- exude the spirit of a new era. Here Hausdorff presents for the first time, beginning with his axioms for neighborhoods, a systematic theory of topological spaces, to which he added the separation axiom known today by his name. This theory arose through a comprehensive synthesis involving the work of other mathematicians as well as his own reflections on the space problem.
Hausdorff created a number of fundamentally new constructions for topology, such as the interior and closure operations, while developing the fundamental concepts of open set and compactness, a concept he took from Fréchet. He also established and developed the theory of connectedness, introducing in particular the notions of "components" and "quasi-components."
He further specialized general topological spaces by means of the first and second Hausdorff countability axioms. The metric spaces comprise a large class of spaces that satisfy the first countability axiom. These were introduced in 1906 by Fréchet; the terminology "metrischer Raum" is derived from Hausdorff.
In his *Grundzuge*, he gave a systematic presentation of the theory of metric spaces, to which he added several new concepts (Hausdorff metric, completion, total boundedness, p-connectedness, reducible sets). Fréchet's work (1906) had received little attention; it was through Hausdorff's *Grundzuge* that metric spaces became widely familiar to mathematicians.
Both the chapter on mappings as well as the final chapter of the *Grundzuge* on measure theory and integration are impressive for the generality of their approach and the originality of the presentation. Hausdorff laconic remarks pointing to the significance of measure theory for probability would prove to be highly insightful. The final chapter also contains the first correct proof of Borel's strong law of large numbers. Finally, the appendix contains the single most spectacular result in the whole book, namely, Hausdorff's theorem that one cannot define a finitely additive measure, invariant under congruences, on all bounded subsets in R^n for $n \geqq \mathbf{3}$. Hausdorff's proof is by means of a famous paradoxical decomposition of the sphere, for which it is necessary to invoke the axiom of choice.
The *Grundzüge der Mengenlehre* appeared at the dawn of the First World War. When it broke out, in August 1914, scientific life in Europe was affected in the most dramatic ways. Under these circumstances, Hausdorff's book had little impact for five to six years. After the war, a new generation of researchers began to take up the many suggestive impulses it contained, especially for topology, now a central field of interest. The reception of Hausdorff's ideas was enhanced by the founding in 1920, of a new journal in Poland, *Fundamenta Mathematicae*. This was the first mathematical journal specializing in the fields of set theory, topology, the theory of real functions, measure theory and integration, functional analysis, logic, and the foundations of mathematics. Within this spectrum of interests, general topology occupied a central place. Hausdorff's *Grundzuge* was cited with great frequency beginning with the very first issue of *Fundamenta Mathematicae*. In the 558 articles (excluding the three written by Hausdorff himself) that appeared in the first twenty volumes between 1920 and 1933, no fewer than 88 referred to the *Grundzuge*. Here one must also take account that Hausdorff's concepts had become so commonplace that one finds these in several articles in which he was not explicitly cited.
Hausdorff's *Grundzuge* had a similar influence on the Russian topological school founded by Paul Alexandroff and Paul Urysohn. This is evident from his correspondence with Alexandroff and Urysohn (after Urysohn's early death with Alexandroff alone) as well as from Urysohn's *Mémoire sur les multiplicités Cantoriennes* (1925-1926), a work the size of a book in which Urysohn set forth his theory of dimension, citing the *Grundzuge* no less than sixty times. The demand for Hausdorff's book continued until well after the Second World War, as attested by the three Chelsea reprints that appeared in 1949, 1965, and 1978.

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